ASYNCHRONOUS AND UNSYMMETRIC COUPLED LINES

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By
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CERTIFICATE

This is to certify that the work on 'Asynchronous and Unsymmetric Coupled Lines' has been carried out under my supervision and this has not been submitted elsewhere for a degree.

August, 1976.

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ABSTRACT

Admittance, impedance, scattering and other matrix parameters are derived for designing a pair of coupled lines of unequal widths embedded in a non-homogenous dielectric medium, a case termed as 'Asynchronous and Unsymmetric'.

Conditions for perfect matching and directivity for asynchronous and unsymmetric pair of coupled lines are derived. These conditions are used to obtain the relations between the design parameters are line parameters.

Equivalent circuits and A B C D parameters have been reported for various filter configurations obtained by putting different end conditions at two of the four ports.

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LIST OF SYMBOLS

V	Voltage on the lines
V_1, V_2, V_3, V_4	Voltages on four ports
I	Current on the lines
I_1, I_2, I_3, I_4	Currents on four ports
z_1, z_2	Line impedances for 1 and 2
z_m	Mutual impedance between lines 1 and 2
y_1, y_2	admittances for linesl and 2
y_m	mutual admittance between lines 1 and 2
'c', 'π'	subscripts used for two modes of propogation
γ	propogation constant
R	ratio of the voltages on two lines
R_3	defines the condition for congurence
Y_{c1}, Y_{c2}	Characteristic admittances of the two lines in 'c' mode
$Y_{\pi1}, Y_{\pi2}$	Characteristic admittances of the two line in 'π' mode
Z_{c1}, Z_{c2}	Characteristic impedances of the two lines in 'c' mode
$Z_{\pi1}, Z_{\pi2}$	Characteristic impedances of the two lines in 'π' mode
A_1, A_2, A_3, A_4	Amplitude coefficients in voltage and current relation
[Y]	admittance matrix of a pair of coupled lines having sixteen elements
[Z]	4x4 impedance matrix for a pair of coupled lines
Z_o	Characteristic impedance of the line
γ_{cl}	is equal to $\gamma_c \cdot l$
$\gamma_{\pi l}$	is equal to $\gamma_\pi \cdot l$

θ_c	electrical length of the lines in 'c' mode
θ_π	electrical length of the lines in 'π' mode
$[S]$	scattering matrix for a coupled line system with 16 entries
E_1	E.M.F. of the generator put on port 1
E_2	E.M.F. of the generator put on port 2
α	$Z_o^a/\Delta Z_o^b$, ratio of the impedance of two lines
v_c	propogation velocity of 'c' mode
v_π	propogation velocity of 'π' mode
$\angle$	phase angle associated
W	width of the line
h	height of the substrate
ε_r	relative dielectric constant of the substrate
S	gap width between the two lines
$C_{sc}\theta$	Cosecant of angle θ

ERRATA

Page	
2	Line 3 - delete 'shown in Fig. 1(a)'.
33	Ref [6] to Ref [7].
4	Last line -'proximation' to approximation
6	Line 2 'lives' to 'lines'
7	Line 4 (1.1) to (2.1)
13	Reference Nos. [10] in 3rd and 3rd last line
14	Eqn. 2.23 $T_{\pi 1}$ to $Y_{\pi 1}$
15	After eqn. 2.28 add 'where $Z = z_1 z_2 - z_m^2$',
17	Eqn. (2.41) and (2.42) γ_i to γ_c
	'The above results ......' 'are seen'
20	Eqn. 3.8 'R_2' to 'R_3'
22	'Internal incident ......' are in negative direction
28	Line 5 'two' to 'to'
	Line 7 delete 'there be'
30	2nd last line 'E_3' to 'E_2'
33	'expressions' to 'expression'
	Line 10 'expression' to 'expressions for'
	Line 11 'expression' to 'expressions'
35	(3.73) to (3.74)
40	K, G to K and G
54	(2) to (ii) in line 4
	Line 3 'gives' to 'shows'
57	2nd line (..........) of
59	'where a,b,c,d,e and f are.......'
81	For 'MIC' read 'microwaves'
	Also for 'Propogation' read 'propagation', for 'homogenous' read 'homogeneous' and for 'congurent' read 'congruent'.

Chapter 1

INTRODUCTION

The mechanism and theory of electro magnetic coupling between parallel high frequency transmission lines sharing the medium where the propogation takes place, has been the object of studies for many decades. The theory has been developed for both arbitrary lines in a homogenous dielectric medium as well as of symmetric lines embeded in an inhomogenous dielectric medium, not to mention of the simplest case of symmetric equal lines in a homogenous dielectric medium.

Besides the fundamental academic interest of the subject, in the context of network analysis and synthesis, there are many practical applications of coupled lines. Uniform coupled line circuits are used for many applications such as filters, directional couplers and impedance matching networks. These circuits are usually designed by utilizing the impedance, admittance, chain and other parameters characterising the coupled line four port network.

We will be restricting our discussions to a system of two parallel coupled lines, sustaining TEM waves. This represents a wide variety of more general situations. First and simplest of these cases is a pair of symmetric coupled lines in a homogeneous dielectric medium, i.e., two

conductors placed close to each other in a uniform dielectric medium like in case of a symmetric pair of coupled striplines shown in Fig. 1(a). Secondly when the medium is isotropic or homogenous, but the two lines are of unequal widths and are not symmetric with respect to a longitudinal plane half way between the two lines. For example, a pair of coupled striplines of unequal strip widths. Thirdly, when the dielectric medium itself is not uniform or homogenous but the coupled lines are symmetric. This case is similar to a symmetric pair coupled microstrips. Fourth and the last of these cases is the most general one where neither the dielectric medium is homogenous nor the two coupled lines are symmetrical, i.e. a pair of unequal coupled microstrip lines. Such a case is termed as 'Unsymmetric and Asynchronous' and will be dealt here.

Solutions for many of the cases of coupled wave propagation mentioned above are already known. E.M.T. Jones and Balljahn [2] has given solutions for the first of the cases discussed above, a pair of coupled lines with line-to-line symmetry with respect to a longitudinal plane half way between the equal lines in a homogenous medium, using the method of analysis developed by Reed and Wheeler [1] for longitudinally symmetric four ports. This analysis yielded

the four distinct entries of the Z-matrix of a symmetric coupled line four port, on the basis of so called even and odd mode impedances, Z_{oe} and Z_{oo}. In the frame of this theory, the electromagnetic coupling of waves propogating along coupled lines, is described in terms of linear superposition of two uncoupled waves, the even and odd mode waves, moving on both lines at the same propogation velocity but at different impedance levels.

The similar analysis was later extended to the cases of unequal lines by Ozaki and Ishii [6] and later by Cristal [3]. This resulted in six distinct entries of the unsymmetric coupled lines four port Z-matrix on the basis of Z^a_{oe} , Z^a_{oo} and Z^b_{oo} and Z^b_{oe}, the even and odd mode impedances of lines 'a' and 'b' respectively. Once again, two modes are characterized by one and the same propogation velocity and same electrical length θ .

With the introduction of microwave hybrid technology in last few years the activity has been shifted to theory of coupled line systems with a non homogenous propogation medium, in which non homogenity occurs in a cross-section orthogonal to the direction of wave propogation [7], [8]. In case of equal lines on an inhomogenous dielectric medium

(symmetric Asynchronous Coupled lines) the even and odd mode analysis can still be applied and one obtains four distinct entries of four port impedance matrix in the same way as for the homogenous dielectric medium case, with two propogation velocities for two different modes instead of one as in the case for homogenous medium. Thus the four port matrix is characterized by two linearly dependent electrical lengths θ_e and θ_o. Such cases where the two propogation velocities are different or $\theta_o \neq \theta_e$ are called as 'Asynchronous'.

This thesis is intended for development of generalized formulae suited to the situation of a pair of unsymmetric parallel coupled lines embedded in a non-homogenous dielectric medium, as those used in technique of suspended substrate striplines or microstriplines of unequal widths on a dielectric medium, following the geometry shown in Fig. 1. Keeping in view the activity and usefulness of Asynchronous and unsymmetric coupled lines, this topic has been chosen for further work. One distinct advantage, which an unsymmetric pair of coupled lines has over the symmetric lines is that it combines two functions, coupling and impedance transformation, in one, unlike in the later. The presence of suspended substrate prevents, strictly speaking, a wave propogation according to purely TEM wave mode; it is known however, that this kind of proximation yields good results in many practical situation.

Recalling that even and odd modes of excitation correspond to the cases where the voltages and currents are equal in magnitude and in phase for even mode and out of phase for the odd mode, it is seen that such modes can not propogate independently for the case of asymmetric coupled lines [9]. Speciable has reported some experimental results [10] where it is observed that a very interesting and simple condition exists in the case of non-symmetrical parallel coupled lines in a non-homogenous medium which reduces the fundamental modes to a voltage even mode and a current odd mode. This is true for unsymmetric and asynchronous coupled lines satisfying a 'congurent' symmetry condition given later. This has resulted in a great simplification in the expressions for mode admittance and impedance parameters for the two lines.

We have obtained the mode parameters of a general unsymmetric and Asynchronous coupled line four port in terms of the line properties for two independent modes of excitations. These modes correspond to a linear combination of voltages and currents on two lines which are related in magnitude and phase through terms involving line constants. Scattering parameters for a special case with congurence symmetry have been calculated. Based upon these a condition has been found

for perfect matching and directivity of a pair of Unsymmetric and Asynchronous coupled lives. From this, relations between the design parameters and the line parameters of a pair of unsymmetric and asynchronous coupled lines are derived.

Parameters for various filter configurations which can be obtained using the four port coupled line structure are also derived. Equivalent circuits for most of these configurations are also obtained.

CHAPTER 2

DERIVATION OF ZADMITTANCE MATRIX

The behaviour of an arbitrary system of n parallel coupled lines, propogating sine wave signals, can be represented by following equations :

$$\frac{\partial}{\partial X}[V] = [Z][I] \qquad (1.1)$$

$$\frac{\partial}{\partial X}[I] = [Y][V] \qquad (2.2)$$

where [V] and [I] column matrices whose elements are the symbolic voltages and currents and [Z] and [Y] matrices are the n^{th} order square matrices with elements representing self and mutual impedances and admittances. For a system of 2 Asymmetric lines embedded in a non homogeneous medium (Fig. 1) the behaviour is described by the following set of general equations :

$$-\frac{dv_1}{dx} = z_1 i_1 + z_m i_2 \qquad (2.3)$$

$$-\frac{dv_2}{dx} = z_m i_1 + z_2 i_2 \qquad (2.4)$$

$$-\frac{di_1}{dx} = y_1 v_1 + y_m v_2 \qquad (2.5)$$

$$-\frac{di_2}{dx} = y_2 v_2 + y_m v_1 \qquad (2.6)$$

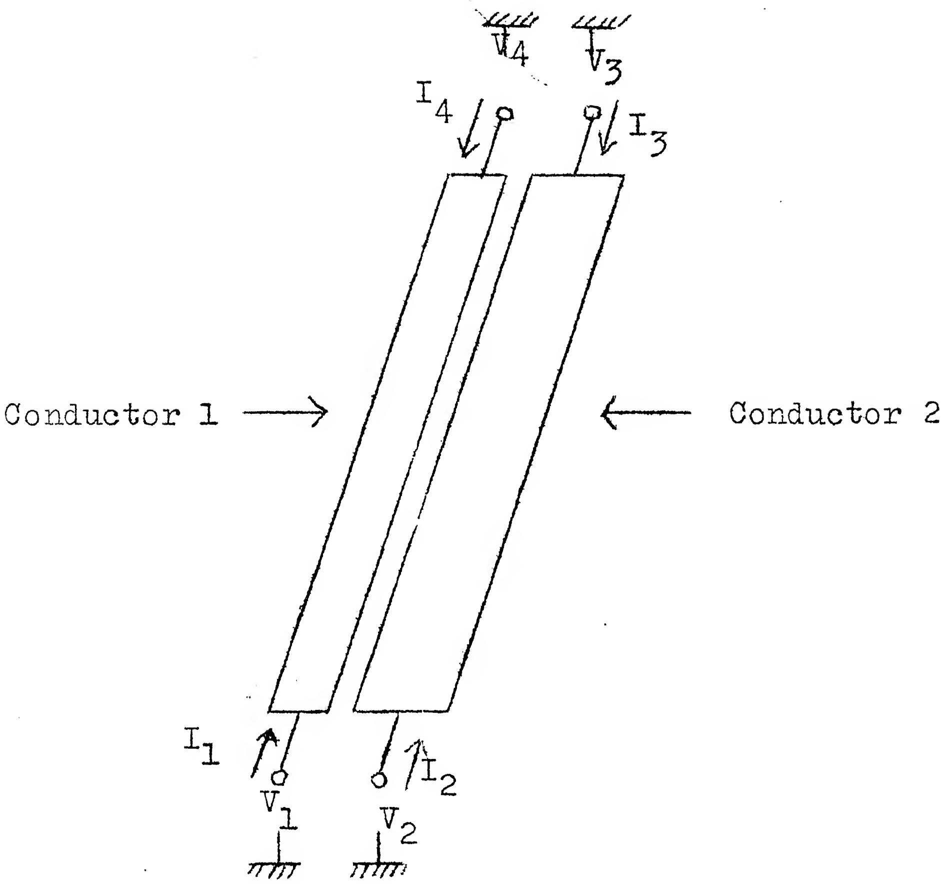

Fig. 1 : A pair of Asymmetric coupled lines in an inhomogenous medium

where z_1 and z_2 are self impedances and y_1 and y_2 are the self admittances per unit length of lines 1 and 2 in the presence of the other line. Also z_m and y_m are mutual impedance and admittance per unit length respectively with an e^{jwt} time variation.

Actually, the equations (2.3) to 2.6) represent voltage and current relationships for any pair of coupled line systems described earlier, i.e. symmetric or unsymmetric and synchronous or Asynchronous pair of coupled lines. In the four cases mentioned earlier, the line parameters z_1, z_2, z_m or y_1, y_2 and y_m satisfy different conditions.

Differentiating (2.3) and (2.4) and substituting (2.5) and (2.6) we get a system of equations for voltages on uniformly coupled lines

$$\frac{d^2v_1}{dx^2} - a_1v_1 - b_1v_2 = 0 \tag{2.7}$$

$$\frac{d^2v_2}{dx^2} - a_2v_2 - b_2v_1 = 0 \tag{2.8}$$

where

$$a_1 = y_1z_1 + y_mz_m$$

$$a_2 = y_2z_2 + y_mz_m$$

$$b_1 = z_1y_m + y_2z_m$$

$$b_2 = z_2y_m + y_1z_m \tag{2.9}$$

Assuming a voltage variation of the type $v(x) = v_o e^{\gamma x}$, because none of the coefficients vary with x, we get the solution of resulting eigen value problem. It is to be noticed here that in a non homogenous dielectric medium the roots of the characteristic equation of the above system will be all different even without losses. The resulting value of four different roots of γ, corresponding to in-phase and antiphase waves for a class of lossless lines. These roots are :

$$\gamma_{1,2} = \pm \gamma_c$$

and

$$\gamma_{3,4} = \pm \gamma_\pi$$

where

$$\gamma^2_{c,\pi} = \frac{a_1+a_2}{2} \pm \tfrac{1}{2}[(a_1-a_2)^2 + 4\ b_1 b_2]^{\frac{1}{2}} \qquad (2.10)$$

From these we obtain two propogation velocities.

It has been observed experimentally that both the modes travel at the same velocity of propogation in a homogenous medium while there are two different velocities in case of an inhomogenous medium. A possible explanation for these can be given as below. In an homogenous dielectric medium both the modes of propogation are subject to same conditions by the dielectric medium and relative dielectric constant is same in both the modes. Assuming the TEM mode of propogation, this indicates that there is only one velocity

of propogation for the two modes. Thus, $(a_1-a_2)^2 + 4\, b_1 b_2 = 0$ and $\sqrt{(a_1+a_2)}/2 = w/v$, where v is the velocity of light in the dielectric medium. For identical lines the above condition further implies $k_y = k_z$ or $k_z = -k_y$ where $k_y = y_m/\sqrt{y_1 \cdot y_2}$ and $k_z = z_m/\sqrt{z_1\ z_2}$. When the dielectric medium is inhomogenous, the two modes are affected in a different way by the dielectric medium. In one of the modes having signals in opposite phase on two coupled lines, there is a larger propotion of the electric field outside the dielectric medium. For example, in case of microstrip coupled lines, upon odd mode excitation, more electric field is outside the dielectric than during even mode excitation. Thus there are two effective dielectric constants for two different mode propogating on coupled lines. In such a case, $(a_1-a_2)^2 + 4\, b_1 b_2 \neq 0$ and more than one phase velocity will occur. In general, $k_z \neq \pm k_y$ this situation is referred to as 'co-directional coupling' since forward waves (+ modes) can only couple to forward wave and backward waves can only couple with backard wave. For the case of $k_z = k_y$, it is referred to as 'contradirectional' coupling. It can be shown that, for homogenous medium $k_z = -k_y$ and that $k_z = k_y$ can only be obtained by adding excess capacitance to the lines[15].

Let us define here the ratios of voltages on two lines for each of the waves,this is given by :

$$\frac{v_2}{v_1} = \frac{\gamma^2 - a_1}{b_1} = \frac{b_2}{\gamma^2 - a_2} \tag{2.11}$$

or

$$R_c \triangleq \frac{v_2}{v_1} \quad \text{for} \quad \gamma = \pm \gamma_c$$

$$= \frac{1}{2b_1}\left\{(a_2 - a_1) + [(a_2-a_1)^2 + 4\, b_1 b_2]^{\frac{1}{2}}\right\} \tag{2.12}$$

and $$R\pi \triangleq \frac{v_2}{v_1} \quad \text{for} \quad \gamma = \pm \gamma_\pi$$

$$= \frac{1}{2b_1}\left\{(a_2-a_1) - [(a_2-a_1)^2 + 4b_1 b_2]^{\frac{1}{2}}\right\} \tag{2.13}$$

From these one notices that $\frac{v_2}{v_1}$ is positive real for one mode and negative real for other mode for a large class of lossless coupled line system where $b_1 b_2 > 0$. In case of identical lines $R_c = +1$ and $R_\pi = -1$ and two modes are same as even and odd mode defined normally.

One very interesting result comes into view if in equations (2.9)

$$a_1 + b_1 = a_2 + b_2 \tag{2.14}$$

or $$a_1 - a_2 = b_2 - b_1 \tag{2.15}$$

or

$$\frac{y_1 + y_m}{y_2 + y_m} = \frac{z_2 - z_m}{z_1 - z_m} \triangleq R_3 \tag{2.16}$$

In this case, which is called congurence [10] we get

$$R_c = \frac{1}{2b_1} \left\{ (a_2-a_1) + [(b_2-b_1)^2 + 4\ b_1 b_2]^{\frac{1}{2}} \right\}$$

$$= \frac{1}{2b_1} \quad [b_1-b_2 + (b_2+b_1)]$$

$$R_c = +1 \tag{2.17}$$

and $$R_\pi = - \frac{y_1 + y_m}{y_2 + y_m} = -R_3 \tag{2.18}$$

The corresponding ratios of current on the two lines are

$$\frac{i_2}{i_1} = \frac{1}{R_3} \quad \text{for} \quad \gamma = +\gamma_c$$

$$\frac{i_2}{i_1} = -1 \qquad \text{for } \gamma \pm \gamma_\pi \tag{2.20}$$

This situation has actually been experimentally verified by speciale[10] with redefined even and odd mode definitions. He has also gone ahead in calculating admittance matrix based upon these definitions.

For finding out the admittance matrix of a generalized case we write down the general solution of voltages on the two lines in terms of all the four waves found earlier. This is given by

$$v_1 = A_1 e^{-\gamma_c x} + A_2 e^{\gamma_c x} + A_3 e^{-\gamma_\pi x} + A_4 e^{\gamma_\pi x} \qquad (2.21)$$

$$v_2 = A_1 R_c e^{-\gamma_c x} + A_2 R_c e^{\gamma_c x} + A_3 R_\pi e^{-\gamma_\pi x} + A_4 R_\pi e^{\gamma_\pi x} \qquad (2.22)$$

The corresponding currents can be obtained by substituting the above values in equations (2.3) and (2.4). These are

$$i_1 = A_1 Y_{c1} e^{-\gamma_c x} - A_2 Y_{c1} e^{\gamma_c x} + A_3 Y_{\pi 1} e^{-\gamma_\pi x} - A_4 T_{\pi 1} e^{\gamma_\pi x} \qquad (2.23)$$

$$i_2 = A_1 R_c Y_{c2} e^{-\gamma_c x} - A_2 R_c Y_{c2} e^{\gamma_c x} + A_3 R_\pi Y_{\pi 2} e^{-\gamma_\pi x} - A_4 R_\pi Y_{\pi 2} e^{\gamma_\pi x} \qquad (2.24)$$

The direction of currents is as shown in Fig. 1. And Y_{c1}, Y_{c2}, $Y_{\pi 1}$ and $Y_{\pi 2}$ are the characteristic admittances of lines 1 and 2 for the two modes and these are given by

$$Y_{c1} = \gamma_c \frac{z_2 - z_m R_c}{Z} = \frac{1}{Z_{c1}} \qquad (2.25)$$

$$Y_{c2} = \frac{\gamma_c}{R_c} \frac{z_1 R_c - z_m}{Z} = \frac{1}{Z_{c2}} \qquad (2.26)$$

$$Y_{\pi 1} = \gamma_\pi \frac{z_2 - z_m R_\pi}{Z} = \frac{1}{Z_{\pi 1}} \tag{2.27}$$

$$Y_{\pi 2} = \frac{\gamma_\pi}{R_\pi} \frac{z_1 R_\pi - z_m}{Z} = \frac{1}{Z_{\pi 2}} \tag{2.28}$$

From these and equations (2.12) and (2.13) for R_c and R_π respectively it is seen that

$$\frac{Y_{c1}}{Y_{c2}} = \frac{Y_{\pi 1}}{Y_{\pi 2}} = -R_c R_\pi \tag{2.29}$$

From the equations given above for voltages and current, we get

$$[I] = [Y] [A] \tag{2.30}$$

and

$$[V] = [Z] [A] \tag{2.31}$$

where $[I]$ and $[V]$ are current and voltage column matrices respectively and $[A]$ is a column matrix of 4 constants A_1, A_2, A_3 and A_4. From these we can get admittance matrix by writing current matrix in terms of voltage matrix and eliminating amplitude coefficients $[A]$. Thus we get

$$[I] = [Y] [V] \tag{2.32}$$

where the 16 entries of the Y-matrix for 4 ports are as given below :

$$Y_{11} = Y_{44} = \frac{Y_{c1} \text{ Coth } \gamma_{c1}}{(1 - R_\pi/R_c)} + \frac{Y_{\pi 1} \text{ Coth } \gamma_{\pi 1}}{(1 - R_c/R_\pi)} \tag{2.33}$$

$$Y_{12} = Y_{21} = Y_{34} = Y_{43} = - \frac{Y_{c1} \text{ Coth } \gamma_{c1}}{R_\pi(1-R_c/R_\pi)} - \frac{Y_{\pi 1} \text{ Coth } \gamma_{\pi 1}}{R_c(1-R_\pi/R_c)} \tag{2.34}$$

$$Y_{13} = Y_{31} = Y_{24} = Y_{42} = \frac{Y_{c1}}{(R_\pi - R_c) \text{Sinh } \gamma_{c1}} + \frac{Y_{\pi 1}}{(R_c - R_\pi) \text{Sinh } \gamma_{\pi 1}} \tag{2.35}$$

$$Y_{14} = Y_{41} = - \frac{Y_{c1}}{(1-R_c/R_\pi) \text{Sinh } \gamma_{c1}} - \frac{Y_{\pi 1}}{(1-R_\pi/R_c) \text{Sinh } \gamma_{\pi 1}} \tag{2.36}$$

$$Y_{22} = Y_{33} = - \frac{R_c \; Y_{c2} \text{ Coth } \gamma_{c1}}{(R_\pi - R_c)} - \frac{R_\pi \; Y_{\pi 2} \text{ Coth } \gamma_{\pi 1}}{(R_c - R_\pi)} \tag{2.37}$$

$$Y_{23} = Y_{32} = \frac{R_c \; Y_{c2}}{(R_\pi - R_c) \text{ Sinh } \gamma_{c1}} + \frac{R_\pi \; Y_{\pi 2}}{(R_c - R_\pi) \text{ Sinh } \gamma_{\pi 1}} \tag{2.38}$$

Similarly one can obtain 6 independent entries of a four port Z-matrix by obtaining [V] in terms of [Z] and [I] and eliminating amplitude coefficients [A] in (2.30) and (2.31).

The elements of 4 x 4 Z-matrix are given by :

$$Z_{11} = Z_{44} = \frac{Z_{c1} \text{ Coth } \gamma_{c1}}{(1 - R_c/R_\pi)} + \frac{Z_{\pi 1} \text{ Coth } \gamma_{\pi 1}}{(1 - R_\pi/R_c)} \tag{2.39}$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = \frac{Z_{c1}\ R_c\ \mathrm{Coth}\ \gamma_{c1}}{(1 - R_c/R_\pi)} + \frac{Z_{\pi 1}\ R_\pi\ \mathrm{Coth}\ \gamma_{\pi 1}}{(1 - R_\pi/R_c)}$$

$$= - \frac{Z_{c2}\ \mathrm{Coth}\ \gamma_{c1}}{R_\pi(1 - R_c/R_\pi)} - \frac{Z_{\pi 2}\ \mathrm{Coth}\ \gamma_{\pi 1}}{R_c(1 - R_\pi/R_c)} \qquad (2.40)$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = \frac{R_c\ Z_{c1}}{(1-R_c/R_\pi)\ \mathrm{Sinh}\ \gamma_i l} + \frac{Z_{\pi 1}}{(1-R_\pi/Rc)\mathrm{Sinh}\ \gamma_{\pi 1}}$$

$$(2.41)$$

$$Z_{14} = Z_{41} = \frac{Z_{c1}}{(1-R_c/R_\pi)\mathrm{Sinh}\ \gamma_i l} + \frac{Z_{\pi 1}}{(1-R_\pi/R_c)\mathrm{Sinh}\ \gamma_\pi l} \qquad (2.42)$$

$$Z_{22} = Z_{33} = - \frac{R_c\ Z_{c2}\ \mathrm{Coth}\ \gamma_{c1}}{R_\pi\ (1-R_c/R_\pi)} - \frac{R_\pi\ Z_{\pi 2}\ \mathrm{Coth}\ \gamma_{\pi 1}}{R_c(1-R_\pi/R_c)} \qquad (2.43)$$

$$Z_{23} = Z_{32} = \frac{R_c^{\ 2}\ Z_{c1}}{(1-R_c/R_\pi)\mathrm{Sinh}\ \gamma_{c1}} + \frac{R_\pi^{\ 2}\ Z_{\pi 1}}{(1-R_\pi/R_c)\mathrm{Sinh}\ \gamma_{\pi 1}} \qquad (2.44)$$

The above results, as expected, seen to be reducing to more familiar values of some special cases. For example, in case of symmetric coupled lines where $y_1 = y_2 = y$; $z_1 = z_2 = z$. Then $R_c = 1$ and $R_\pi = -1$, we get :

$Z_{c2} = Z_{c1} = Z_{oe}$ the even mode impedance and $Z_{\pi 1} = Z_{\pi 2} = Z_{oo}$ the odd mode impedance with :

$$\gamma_{c,\pi} = [(y \pm y_m)\ (z \pm z_m)]^{\frac{1}{2}} = \gamma_{e,o} \qquad (2.45)$$

and resulting expression of 4-port matrix parameters are same as those in Zysman and Johnson [4] for an inhomogenous medium and Jones and Bolljahn [2] for a homogenous medium (for TEM case $y_m/y = - z_m/z$).

The results also match with Speciale's [10] for concurrent symmetry case obtained with the new definition of even and odd modes. For example, substituting $R_c = 1$ and $R_\pi = -R_3$ in equations (1.33) to (1.38) we obtain :

$$Y_{11} = Y_{44} = [R_3 Y_{c1} \text{ Coth } \gamma_{c1} + Y_{\pi 1} \text{ Coth } \gamma_{\pi 1}]/(1+R_3) \tag{2.46}$$

$$Y_{12} = Y_{21} = Y_{34} = Y_{43} = [Y_{c1} \text{ Coth } \gamma_{c1} + Y_{\pi 1} \text{ Coth } \gamma_{\pi 1}]/(1+R_3) \tag{2.47}$$

$$Y_{13} = Y_{31} = Y_{24} = Y_{42} = [-Y_{c1}/\text{Sinh } \gamma_{c1} + Y_{\pi 1}/\text{Sinh } \gamma_{\pi 1}]/(1+R_3) \tag{2.48}$$

$$Y_{14} = Y_{41} = -[R_3 Y_{c1}/\text{Sinh } \gamma_{c1} + Y_{\pi 1}/\text{Sinh } \gamma_{\pi 1}]/(1+R_3) \tag{2.49}$$

$$Y_{22} = Y_{33} = [-Y_{c2} \text{ Coth } \gamma_{c1} + R_3 Y_{\pi 2} \text{ Coth } \gamma_{\pi 1}]/(1+R_3) \tag{2.50}$$

$$Y_{23} = Y_{32} = [Y_{c2}/\text{Sinh } \gamma_{c1} + R_3 Y_{\pi 2}/\text{Sinh } \gamma_{\pi 1}]/(1+R_3) \tag{2.51}$$

These results are similar to those obtained by Specialle [10]for lossless coupled lines with congurent symmetry.

CHAPTER 3

DERIVATION OF SCATTERING MATRIX

The scattering matrix for a generalized case of Asymmetric coupled lines in an inhomogeneous medium can be derived using the Admittance or Impedance matrices obtained earlier. But this procedure becomes quite complicated and involved.

A step by step procedure has been applied to find out the scattering parameters of a system of coupled line which satisfy congurence condition given by equation (2.16). Once again for such a case $R_c = +1$ and $R_\pi = -R_3$. Taking the reference origin of co-ordinate system at port 4 in Fig. 1 with positive semi axis pointing towards port 1, we get :

$$V_1 = A_1 e^{j\theta_c} + A_2 e^{-j\theta_c} + A_3 e^{j\theta_\pi} + A_4 e^{-j\theta_\pi} \tag{3.1}$$

$$V_2 = A_1 e^{j\theta_c} + A_2 e^{-j\theta_c} - R_3 A_3 e^{j\theta_\pi} - R_3 A_4 e^{-j\theta_\pi} \tag{3.2}$$

$$V_3 = A_1 + A_2 - R_3 A_3 - R_3 A_4 \tag{3.3}$$

$$V_4 = A_1 + A_2 + A_3 + A_4 \tag{3.4}$$

where $\quad \mathrm{Coth}\ \gamma_{c1} = -j\ \mathrm{Cot}\ \theta_c$

$\quad \mathrm{Coth}\ \gamma_{\pi 1} = -j\ \mathrm{Cot}\ \theta_\pi$

where θ_c and θ_π are the electrical lengths

Λ_1 = even mode incident (forward) wave

Λ_2 = even mode reflected (backward) wave

Λ_3 = odd mode incident wave

and Λ_4 = odd mode reflected wave

Subtracting (3.2) from (3.1) and (3.3) from (3.4)

$$V_1 - V_2 = (1+R_3)\, e^{j\theta_\pi} \Lambda_3 + (1+R_3)\, e^{-j\theta_\pi} \Lambda_4 \tag{3.5}$$

and

$$V_4 - V_3 = (1+R_3)\, \Lambda_3 + (1+R_3)\, \Lambda_4 \tag{3.6}$$

again [eqn. 3.2 + R_3 x eq. 3.1] and [Equation 3.3 + R_3x3.4] we get

$$R_3 V_1 + V_2 = (1+R_3)\, e^{j\theta_c} \Lambda_1 + (1+R_3)\, e^{-j\theta_c} \Lambda_2 \tag{3.7}$$

$$R_3 V_4 + V_3 = (1+R_3)\, \Lambda_1 + (1+R_3)\, \Lambda_2 \tag{3.8}$$

From (3.5) and (3.6) we get Λ_3 and Λ_4 and from (3.7) and (3.8) we get Λ_1 and Λ_2:

These are

$$\Lambda_1 = \frac{1}{1 + R_3} \; \frac{\begin{vmatrix} R_3 V_1 + V_2 & e^{-j\theta_c} \\ R_3 V_4 + V_3 & 1 \end{vmatrix}}{\begin{vmatrix} e^{j\theta_c} & e^{-j\theta_c} \\ 1 & 1 \end{vmatrix}}$$

$$= \frac{1}{1 + R_3} \; \frac{(R_3 V_1 + V_2) - (R_3 V_4 + V_3)\, e^{-j\theta_c}}{e^{j\theta_c} - e^{-j\theta_c}}$$

$$A_1 = \frac{1}{1 + R_3} \cdot \frac{1}{2j \text{ Sin } \theta_c} \left[(R_3 V_1 + V_2) - (R_3 V_4 + V_3)\, e^{-j\theta_c} \right] \tag{3.9}$$

Similarly,

$$A_2 = - \frac{1}{1+R_3} \; \frac{1}{2\, j \text{ Sin } \theta_c} \left[(R_3 V_1 + V_2) - (R_3 V_4 + V_3)\, e^{j\theta_c} \right] \tag{3.10}$$

and

$$A_3 = \frac{1}{1+R_3} \; \frac{1}{2j \text{ Sin } \theta_\pi} \left[(V_1 - V_2) - (V_4 - V_3)\, e^{-j\theta_\pi} \right] \tag{3.11}$$

$$A_4 = \frac{1}{1+R_3} \; \frac{1}{2j \text{ Sin } \theta_\pi} \left[(V_1 - V_2) - (V_4 - V_3)\, e^{j\theta_\pi} \right] \tag{3.12}$$

Defining the four port currents I_1, I_2, I_3 and I_4 by multiplying with appropriate even and odd mode line admittances and impedances .

$$I_1 = Y_{c1} A_1 e^{j\theta_c} + \frac{A_3}{Z_{\pi 1}} e^{-j\theta_\pi} - Y_{c1} A_2 \exp(-j\theta_c) - \frac{A_4}{Z_{\pi 2}} \exp(-j\theta_\pi) \tag{3.13}$$

$$I_2 = Y_{c2}\ A_1\ \exp(j\theta_c) - \frac{R_3 A_3}{Z_{\pi 2}} \exp(j\theta_\pi) - Y_{c2}\ A_2\ \exp(-j\theta_c)$$

$$+ R_3 \frac{A_4}{Z_{\pi 2}} \exp(-j\theta_\pi) \tag{3.14}$$

$$I_3 = -Y_{c2}A_1 + R_3 \frac{A_3}{Z_{\pi 2}} + Y_{c2}\ A_2 - R_3 A_4 / Z_{\pi 2} \tag{3.15}$$

$$I_4 = -Y_{c1}A_1 - \frac{A_3}{Z_{\pi 1}} + Y_{c1}\ A_2 + \frac{A_4}{Z_{\pi 2}} \tag{3.16}$$

Internal incident waves on ports 1 and 2 are positive and on ports 3 and 4 are negitive. Converse is true for internal reflected waves. If the corresponding wave voltage is positive which is not the case for odd mode voltage in line 2, hence reversal of sign. From (3.13) to (3.16) we get $(I_1 + I_2)$, $(I_4 + I_3)$ and $(I_1 - R_3 I_2)$, $(I_4 - R_3 I_3)$. From these we get the values of A_1, A_2 and A_3, A_4 respectively. These expressions come out to be

$$A_1 = K_1[(I_1+I_2) + (I_4+I_3)\ \exp(-j\theta_c)] \tag{3.17}$$

$$A_2 = K_1[(I_1+I_2) + (I_4+I_3)\ \exp(j\theta_c)] \tag{3.18}$$

$$A_3 = K_2[(I_1-R_3 I_2) + (I_4-R_3 I_3)\ \exp(-j\theta_\pi)] \tag{3.19}$$

$$A_4 = K_2[(I_1-R_3 I_2) + (I_4-R_3 I_3)\ \exp(j\theta_\pi)] \tag{3.20}$$

where

$$K_1 = \frac{R_3}{1+R_3}\ \frac{1}{2j\ \mathrm{Sin}\ \theta_c} \cdot \frac{1}{Y_{c1}}$$

and $$K_2 = \frac{Z_{\pi 1}}{1+R_3}\,\frac{1}{2j\ \mathrm{Sin}\ \theta_\pi}$$

This represents value of amplitude coefficients in terms of port currents for a general load condition.

Now we solve from (3.9) to (3.12) and from (3.17) to (3.20) for specific load conditions. Let there be a generator with E.M.F. E_1 and internal impedance Z_o at port 1 and loads of impedance Z_o at other ports then

$$I_1 = (E_1 - V_1)/Z_o \tag{3.21}$$

$$I_2 = -V_2/Z_o \tag{3.22}$$

$$I_3 = -V_3/Z_o \tag{3.23}$$

and $$I_4 = -V_4/Z_o \tag{3.24}$$

Putting these in (3.17) to (3.20) and equating them with (3.9) to (3.12) we get

$$A_1 = M_1[(R_3V_1+V_2) - (R_3V_4+V_3)\ \exp\ (-j\theta_c)]$$

$$= K_1[(E_1-V_1-V_2) - (V_4+V_3)\exp(-j\theta_c)]/Z_o \tag{3.25}$$

$$A_2 = -M_1[(R_3V_1+V_2) - (R_3V_4+V_3)\ \exp(j\theta_c)]$$

$$= K_1[(E_1-V_1-V_2) - (V_4+V_3)\ \exp(j\theta_c)]/Z_o \tag{3.26}$$

$$A_3 = M_2[(V_1-V_2) - (V_4-V_3)\exp(-j\theta_\pi)]$$

$$= K_2[(E_1-V_1+R_3V_2) - (V_4-R_3V_3)\ \exp(-j\theta_\pi)]/Z_o \tag{3.27}$$

$$A_4 = -M_2[(V_1-V_2) - (V_4-V_3)\exp(j\theta_\pi)]$$

$$= K_2[(E_1-V_1+R_3V_2) - (V_4-R_3V_3)\exp(j\theta_\pi)]/Z_o \tag{3.28}$$

where

$$M_1 = \frac{1}{1+R_3}\ \frac{1}{2j\ \text{Sin}\ \theta_c}$$

and

$$M_2 = \frac{1}{1+R_3}\ \frac{1}{2j\ \text{Sin}\ \theta_\pi}$$

From the above equations if we take the terms containing the terms exp $(\pm\ j\theta)$ on one side and common factors are eliminated. We get,

$$(R_3V_1+V_2) - R_3(E_1-V_1-V_2)/y_{c1} = (R_3V_4+V_3)\exp(-j\theta_c)$$

$$- R_3(V_4+V_3)\exp(-j\theta_c)/y_{c1} \tag{3.29}$$

$$-(R_3V_1+V_2) - R_3(E_1-V_1-V_2)/y_{c1} = -(R_3V_4+V_3)\exp(j\theta_c)$$

$$- R_3(V_4+V_3)\exp(j\theta_c)/y_{c1} \tag{3.30}$$

$$(V_1-V_2) - z_{\pi 1}(E_1-V_1+R_3V_2) = (V_4-V_3)\exp(-j\theta_\pi)$$

$$- z_{\pi 1}(V_4-R_3V_3)\exp(-j\theta_\pi) \tag{3.31}$$

$$-(V_1-V_2) - z_{\pi 1}(E_1-V_1+R_3V_2) = -(V_4-V_3)\exp(j\theta_\pi)$$

$$- z_{\pi 1}(V_4-R_3V_3)\exp(j\theta_\pi) \tag{3.32}$$

y_{c1} and $\pi z_{\pi 1}$ are normalized with respect to Z_o. Now adding equations (3.29) and (3.30) and (3.31) and (3.32), we get :

$$-R_3(E_1-V_1-V_2)/y_{c1} = -j \, \mathrm{Sin}\, \theta_c(R_3V_4+V_3) - R_3 \, \mathrm{Cos}\, \theta_c(V_4+V_3)/y_{c1} \tag{3.33}$$

$$R_3V_1 + V_2 = (R_3V_4+V_3) \, \mathrm{Cos}\, \theta_c + j \, R_3 \, \mathrm{Sin}\, \theta_c(V_4+V_3)/y_{c1} \tag{3.34}$$

$$-z_{\pi 1}(E_1-V_1+R_3V_2) = -z_{\pi 1}(V_4-R_3V_3) \, \mathrm{Cos}\, \theta_\pi - j(V_4-V_3)\mathrm{Sin}\, \theta_\pi \tag{3.35}$$

$$V_1-V_2 = j \, z_{\pi 1}(V_4-R_3V_3) \, \mathrm{Sin}\, \theta_\pi + (V_4-V_3) \, \mathrm{Cos}\, \theta_\pi \tag{3.36}$$

From equations (3.33) and 3.34) we can get $(R_3V_4+V_3)$ and (V_4+V_3) in terms of V_1 and V_2 and from equations (3.35) and (3.36) we can get $(V_4-R_3V_3)$ and (V_4-V_3) in terms of V_1 and V_2. These are :

$$(R_3V_4+V_3) = (R_3V_1+V_2) \, \mathrm{Cos}\, \theta_c - j(E_1-V_1-V_3) \, R_3 \, \mathrm{Sin}\, \theta_c/y_{c1} \tag{3.37}$$

$$(V_4+V_3) = (E_1-V_1-V_2) \, \mathrm{Cos}\, \theta_c - jy_{c1}(R_3V_1+V_2) \, \mathrm{Sin}\, \theta_c/R_3 \tag{3.38}$$

$$(V_4-R_3V_3) = (E_1-V_1+R_3V_2) \, \mathrm{Cos}\, \theta_\pi - j(V_1-V_2) \, \mathrm{Sin}\, \theta_\pi/z_{\pi 1} \tag{3.39}$$

$$(V_4-V_3) = (V_1-V_2) \, \mathrm{Cos}\, \theta_\pi - j \, z_{\pi 1}(E_1-V_1+R_3V_2) \, \mathrm{Sin}\, \theta_\pi \tag{3.40}$$

These give us a set of two expressions for V_4 and another set of two expressions for V_3. Equating the expressions for V_4 and V_3 and thereby eliminating V_4 and V_3 we get the equations of the form :

$$A' \cdot V_1 + B' \cdot V_2 = F \cdot E_1 \tag{3.41}$$

$$C' \cdot V_1 + D' \cdot V_2 = H \cdot E_1 \tag{3.42}$$

where

$$A' = -D' = (1+R_3)(\mathrm{Cos}\ \theta_c - \mathrm{Cos}\ \theta_\pi) + j[(R_3/y_{c1}+y_{c1})$$
$$\mathrm{Sin}\ \theta_c - (R_3\ z_{\pi 1}+1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi] \tag{3.43}$$

$$B' = 2(\mathrm{Cos}\ \theta_c+R_3\ \mathrm{Cos}\ \theta_\pi) + j[(R_3/y_{c1}+y_{c1}/R_3)\ \mathrm{Sin}\ \theta_c$$
$$+ (R_3^{\,2}\ z_{\pi 1} + 1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi \tag{3.44}$$

$$C' = -2(R_3\ \mathrm{Cos}\ \theta_c+\mathrm{Cos}\ \theta_\pi) - j[(y_{c1}+1/y_{c1})\ R_3\ \mathrm{Sin}\ \theta_c$$
$$+ (z_{\pi 1}+1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi] \tag{3.45}$$

$$F = (\mathrm{Cos}\ \theta_c - \mathrm{Cos}\ \theta_\pi) + j[R_3\ \mathrm{Sin}\ \theta_c/y_{c1}-R_3 z_{\pi 1}\ \mathrm{Sin}\ \theta_\pi] \tag{3.46}$$

$$H = -(R_3\ \mathrm{Cos}\ \theta_c + \mathrm{Cos}\ \theta_\pi) - j(R_3\ \mathrm{Sin}\ \theta_c/y_{c1} + z_{\pi 1}\ \mathrm{Sin}\ \theta_\pi) \tag{3.47}$$

From this we obtain the port voltages at ports 1 and 2 :

$$V_1 = \frac{\begin{vmatrix} F & B' \\ H & D' \end{vmatrix}}{\begin{vmatrix} A' & B' \\ C' & D' \end{vmatrix}} E_1 = \frac{FD' - HB'}{A'D'-B'C'} E_1$$

$$V_1 = -\frac{FD' - HB'}{A^2 + B'C'} E_1 \tag{3.48}$$

also $$V_2 = \frac{A'H - C'F}{A'D' - B'C'} E_1 = - \frac{A'H - C'F}{A^2 + B'C'} E_1 \qquad (3.49)$$

Now

$$S_{11} = 2\frac{V_1}{E_1} - 1 = 2\frac{FA' + B'H}{A'^2 + B'C'} - 1 \qquad (3.50)$$

and

$$S_{12} = 2\frac{V_2}{E_1} = -2\frac{A'H - C'F}{A'^2 + B'C'} \qquad (3.51)$$

Similarly we can get S_{13} and S_{14} where $S_{13} = 2\ V_3/E_1$ and $S_{14} = 2\ V_4/E_1$. The detailed expression for these two values are given as

$$S_{14} = \frac{2}{R_3-1}\left\{[-1 + (1+R_3)\frac{V_1}{E_1} + \frac{2V_2}{E_1}] \operatorname{Cos} \theta_c - j[\frac{R_3}{y_{c1}}(\frac{R_3}{y_{c1}} + y_{c1})\frac{V_1}{E_1} - (\frac{R_3}{y_{c1}} + \frac{y_{c1}}{R_3})\frac{V_2}{E_1}] \operatorname{Sin} \theta_c\right\} \qquad (3.52)$$

$$= \frac{2}{R_3-1}\left\{[-1 + (1+R_3)\frac{V_1}{E_1} - 2R_3\frac{V_2}{E_1}] \operatorname{Cos} \theta_\pi - j[R_3\ z_{\pi 1} - (R_3\ z_{\pi 1} + \frac{1}{z_{\pi 1}}) \cdot \frac{V_1}{E_1} + (R_3{}^2 z_{\pi 1} + \frac{1.0}{z_{\pi 1}})\frac{V_2}{E_1}] \operatorname{Sin} \theta_\pi\right\} \qquad (3.53)$$

$$S_{13} = \frac{2}{R_3-1}\Bigg\{[R_3-2\ R_3 \frac{V_1}{E_1} -(1+R_3)\frac{V_2}{E_1}]\ \text{Cos}\ \theta_c + j[\frac{R_3}{y_{c1}}$$

$$- (y_{c1} + \frac{1}{y_{c1}}) \cdot R_3 \frac{V_3}{V_1} - (y_{c1} + \frac{R_3}{y_{c1}})\frac{V_2}{E_1}]\ \text{Sin}\ \theta_c]\Bigg\}$$

$$= \frac{2}{R_3-1}\Bigg\{[-1 + \frac{2\ V_1}{E_1} -(1+R_3)\frac{V_2}{E_1}]\ \text{Cos}\ \theta_\pi - j[z_{\pi 1} -(z_{\pi 1} + \frac{1}{z_{\pi 1}})$$

$$\cdot \frac{V_1}{E_1} + (R_3\ z_{\pi 1} + \frac{1}{z_{\pi 1}})\frac{V_2}{E_1}]\ \text{Sin}\ \theta_\pi\Bigg\} \qquad (3.54)$$

The other two impedance quantities in S-matrix which are two be evaluated are parameters S_{22} and S_{23}. For this, let there be a generator of E.M.F. E_2 and internal impedance Z_o be connected at port 2. This gives the following boundary conditions :

$I_1 = -V_1/Z_o$, $I_2 = (E_2-V_2)/Z_o$, $I_3 = -V_3/Z_o$

and $I_4 = -V_4/Z_o$

By putting these in equations (3.17) to (3.20) and equating the same with equations (3.9) to (3.12) respectively we get :

$$A_1 = \frac{1}{1+R_3}\ \frac{1}{2\ j\ \text{Sin}\ \theta_c}\ [(R_3V_1+V_2) -(R_3V_4+V_3)\ \exp(-j\ \theta_c)]$$

$$= \frac{R_3}{1+R_3}\ \frac{1}{Z_oY_{c1}2j\ \text{Sin}\ \theta_c}[(-V_1+E_2-V_2) - (V_4+V_3)\ \exp(-j\theta_c)]$$

(3.55)

and

$$A_2 = -\frac{1}{1+R_3} \cdot \frac{1}{2j \text{ Sin } \theta_c}[(R_3V_1+V_2) - (R_3V_4+V_3) \exp(j\theta_c)]$$

$$= \frac{R_3}{1+R_3} \frac{1}{2j \text{ Sin } \theta_c Y_{cl} Z_o}[(-V_1+E_2-V_2)-(V_4+V_3) \exp(j\theta_c)]$$

(3.56)

$$A_3 = \frac{1}{1+R_3} \quad \frac{1}{2j \text{ Sin } \theta_\pi}[(V_1-V_2)-(V_4-V_3) \exp(-j\theta_\pi)]$$

$$= \frac{z_{\pi 1}}{(1+R_3)\ 2j \text{ Sin } \theta_\pi}[(-V_1-R_3E_2+R_3V_2)-(V_4-R_3V_3) \exp(-j\theta_\pi)]$$

(3.57)

$$A_4 = \frac{1}{1+R_3} \frac{1}{2j \text{ Sin } \theta_\pi} [(V_1-V_3)-(V_4-V_3) \exp (j\theta_\pi)]$$

$$= \frac{z_{\pi 1}}{(1+R_3)\ 2j \text{ Sin } \theta_\pi}[(-V_1-R_3E_2+R_3V_2)-(V_4-R_3V_3) \exp(j\theta_\pi)] \quad (3.58)$$

Once again, we manipulate equations (3.55) to (3.58) to get the following :

$$(R_3V_1+V_2) - \frac{R_3}{y_{cl}}(E_2-V_1-V_2) = (R_3V_4+V_3) \exp(-j\theta_c)$$

$$- R_3(V_4+V_3) \exp(-j\theta_c)/y_{cl} \quad (3.59)$$

$$-(R_3V_1+V_2) - R_3(E_2-V_1-V_2)/y_{cl} = -(R_3V_4+V_3) \exp(-j\theta_c)$$

$$- R_3(V_3+V_4) \exp(j\theta_c)/y_{cl} \quad (3.60)$$

$$(V_1-V_2) + z_{\pi 1}(R_3E_2 + V_1-R_3 V_2) = (V_4-V_3) \exp (-j\theta_\pi)$$

$$-z_{\pi 1}(V_4-R_3V_3) \exp(-j\theta_\pi) \tag{3.61}$$

$$-(V_1-V_2) + z_{\pi 1}(R_3E_2+V_1-R_3V_2) = -(V_4-V_3) \exp(j\theta_\pi)$$

$$- z_{\pi 1}(V_4-R_3V_3) \exp(j\theta_\pi) \tag{3.62}$$

From these we get :

$$-R_3(E_3-V_1-V_2)/y_{c1} = -(R_3V_4+V_3)[\exp(j\theta_c) - \exp(-j\theta_c)]/2$$

$$-R_3(V_4+V_3)[\exp(j\theta_c) - \exp(-j\theta_c)]/2\, y_{c1}$$

$$= -j \operatorname{Sin} \theta_c(R_3V_4+V_3) - R_3 \operatorname{Cos} \theta_c(V_4+V_3)/y_{c1} \tag{3.63}$$

$$(R_3V_1+V_2) = (R_3V_4+V_3) \operatorname{Cos} \theta_c + j\, R_3 \operatorname{Sin} \theta_c(V_4+V_3)/y_{c1} \tag{3.64}$$

$$z_{\pi 1}(R_3E_2+V_1-R_3V_2) = -z_{\pi 1} \operatorname{Cos} \theta_\pi(V_4-R_3V_3) -j \operatorname{Sin} \theta_\pi(V_4-V_3) \tag{3.65}$$

$$(V_1-V_2) = j(V_4-R_3V_3)\, z_{\pi 1} \operatorname{Sin} \theta\pi + (V_4-V_3) \operatorname{Cos} \theta_\pi \tag{3.66}$$

From these we get :

$$V_3 = \frac{1}{(R_3-1)} \Big\{[R_3E_2+2V_1 - (1+R_3)V_2] \operatorname{Cos} \theta_\pi + j[R_3Z_{\pi 1}E_3$$

$$+ (z_{\pi 1}+1/z_{\pi 1})\, V_1-(R_3z_{\pi 1}+1/z_{\pi 1})V_2] \operatorname{Sin} \theta_\pi\Big\}$$

$$= \frac{1}{(R_3-1)} \Big\{ [R_3E_2 - 2R_3V_1 - (1+R_3)V_2] \text{ Cos } \theta_c$$

$$+ j[\frac{R_3}{y_{c1}} E_2 - (y_{c1} + \frac{1}{y_{c1}}) R_3V_1 - (y_{c1} + \frac{R_3}{y_{c1}})V_2] \text{ Sin } \theta_c \Big\} \quad (3.67)$$

$$V_4 = \frac{1}{(R_3-1)} \Big\{ [-E_2 + (1+R_3)V_1 + 2V_2] \text{Cos } \theta_c - j[\frac{R_3}{y_{c1}} E_2 -$$

$$(\frac{R_3}{y_{c1}} + y_{c1}) V_1 - (\frac{R_3}{y_{c1}} + \frac{y_{c1}}{R_3})V_2] \text{ Sin } \theta_c \Big\} \quad (3.68)$$

$$= \frac{1}{(R_3-1)} \Big\{ [R_3E_2 + (1+R_3)V_1 - 2R_3V_2] \text{ Cos } \theta_\pi$$

$$+ j[R_3^2 z_{\pi 1} E_2 + (R_3 z_{\pi 1} + 1/z_{\pi 1})V_1$$

$$-(R_3^2 z_{\pi 1} + 1/z_{\pi 1}) V_2] \text{ Sin } \theta_\pi \Big\}$$

Once again, if we move all terms containing E_2 on right hand side and terms containing V_1 and V_2 on left hand side and eliminating common terms, we get :

$$A'V_1 + B'V_2 = G.E_2$$

$$C'V_1 + D'V_2 = K \cdot E_2$$

where A', B', C' and D' are same as defined earlier in (3.37) to (3.39) and

$$G = (\text{Cos } \theta_c + R_3 \text{ Cos } \theta_\pi) + j\, R_3[\text{ Sin } \theta_c / y_{c1} + R_3 z_{\pi 1} \text{ Sin } \theta_\pi] \quad (3.69$$

$$K = -R_3\left\{(\text{Cos }\theta_c - \text{Cos }\theta_\pi) + j\,[\text{Sin }\theta_c/y_{c1} - z_{\pi 1}\ \text{Sin }\theta_\pi]\right\} \quad (3.70$$

From these we get :

$$V_1 = -\frac{G\ D' - K\ B'}{A'^2 + B'\ C'}E_2 = \frac{G\ A' + K\ B'}{A'^2 + B'C'}\ E_2 \quad (3.7$$

$$V_2 = -\frac{A'\ K - C'G}{A'^2 + B'C'}\ E_2 \quad (3.7$$

again

$$S_{21} = 2\frac{V_1}{E_2} = \frac{2(G\ A' + K\ B')}{A'^2 + B'C'} \quad (3.7$$

and

$$S_{22} = 2\ \frac{V_2}{E_2} - 1 = -2\ \frac{A'K - C'G}{A'^2 + B'C'} - 1 \quad (3.7$$

The other two scattering parameters S_{23} and S_{24} can be calculated by dividing either of the two expressions of V_4 and V_3 in equations (3.67) and (3.68) by E_2 and multiplying by 2. Thus, we get :

$$S_{23} = 2\ \frac{V_3}{E_2} = \frac{2}{R_3-1}\Big\{[(1+R_3)\ V_1/E_2-1 - 2\ V_2/E_2]\ \text{Cos }\theta_c$$

$$-j[R_3/y_{c1} - (R_3/y_{c1} + y_{c1})V_1/E_2$$

$$-(R_3/y_{c1} + y_{c1}/R_3)\ V_2/E_2]\ \text{Sin }\theta_c\Big\}$$

$$= \frac{2}{R_{3-1}}\Big\{[R_3+(1+R_3)V_1/E_2 - 2\ R_3V_2/E_2]\ \text{Cos }\theta_\pi$$

$$+ j\ [R_3^2 z_{\pi 1} + \cancel{(R_3 z_{\pi 1}} + (R_3 z_{\pi 1} + 1/z_{\pi 1})\ V_1/R_3$$

$$- (R_3^2 z_{\pi 1} + 1/z_{\pi 1})\ V_2/R_3]\ \text{Sin }\theta_\pi\Big\} \quad (3.$$

Expressions for S_{24} is same as S_{13} obtained earlier and also it can be shown that S_{21} is same as S_{12} obtained earlier. In all, six independent S-parameters, $S_{11}, S_{12}, S_{13}, S_{14}, S_{22}$ and S_{33} are obtained here.

The expressions for S_{13}, S_{14} and S_{23} become very complex specially when values of V_1/E_1, V_2/E_1 and V_1/E_2, V_2/E_2 are put in the expressions. So based on equation (3.33) to (3.36) an easier solution is tried by eliminating V_1 and V_2 instead of V_3 and V_4 as done earlier from (3.33) and (3.34). Once again, equating the two expression for V_1 and other two expression for V_2 and rearranging the terms with E_1 on one side we get,

$$A'V_4 + B'V_3 = 0$$

$$C'V_4 + D'V_3 = -(1+R_3)\ E_1$$

where A', B', C' and D' are same as defined earlier. From these we get the new values of S_{13} and S_{14}. These are given by

$$S_{13} = \frac{2V_3}{E_1} = \frac{2(1+R_3)A'}{A'^2+B'C'} \tag{3.76}$$

and

$$S_{14} = \frac{2\ V_4}{E_1} = -\frac{2(1+R_3)\ B'}{A'^2 + B'C'} \tag{3.77}$$

Similarly to get an alternative definition of S_{23} we start from equations (3.62) to (3.65) and eliminate V_1 and V_2 instead of V_3 and V_4. This way one gets, after the rearrangement of terms,

$$A'V_4 + B'V_3 = (1+R_3)E_2$$

and

$$C'V_4 + D'V_3 = 0$$

From this we get

$$S_{23} = \frac{2\ V_3}{E_2} = 2\ \frac{(1+R_3)\ C'}{A'^2 + B'C'} \tag{3.78}$$

This way we get all the six independent entries of S-parameter matrix for an asymmetric and unsynchronous pair of coupled lines with congurence. These are summarised below :

$$S_{11} = \frac{2(A'F + B'H)}{DEN} - 1 = S_{44} \tag{3.50}$$

$$S_{12} = \frac{-2(A'H - C'F)}{DEN} = S_{21} = S_{43} = S_{34} \tag{3.51}$$

$$S_{13} = \frac{2(1+R_3)A'}{DEN} = S_{31} = S_{42} = S_{24} \tag{3.76}$$

$$S_{14} = \frac{-2(1+R_3)B'}{DEN} = S_{41} \tag{3.77}$$

$$S_{22} = \frac{-2(A'K - C'G)}{DEN} - 1 = S_{33} \tag{3.73}$$

$$S_{23} = \frac{2(1+R_3)\ C'}{DEN} = S_{32} \tag{3.78}$$

where DEN $= A'^2 + B'C'$ and A', B', C', F, G, H and K are as defined earlier. All these expressions can be simplified in terms of real and imaginary parts.

By finding out expressions for real and imaginary parts in the above we get

$$S_{11} = S_{44} = (X_{1R} + jX_{1I})/DEN \tag{3.79}$$

$$S_{22} = S_{33} = (X_{2R} + jX_{2I})/DEN \tag{3.80}$$

$$S_{12} = S_{21} = S_{43} = S_{34} = (X_{3R} + jX_{3I})/DEN \tag{3.81}$$

$$S_{13} = S_{31} = S_{42} = S_{24} = (X_{4R}+jX_{4I})/DEN \tag{3.82}$$

$$S_{14} = S_{41} = (X_{5R} + jX_{5I})/DEN \tag{3.83}$$

$$S_{23} = S_{32} = (X_{6R} + jX_{6I})/DEN \tag{3.84}$$

$$\text{where DEN} : Y_R + j\ Y_I \tag{3.85}$$

$$\text{and}\quad Y_R = 2\left\{(1-R_3)^2 - [3+2R_3+3R_3^{\,2}]\ \mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi\right\} + [(1+R_3)^2(R_3\ z_{\pi 1}/y_{c1} + y_{c1}/R_3 z_{\pi 1}) + 4R_3/y_{c1}z_{\pi 1} + (1+R_3^{\,2})^2\ y_{c1}z_{\pi 1}/R_3]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi \tag{3.86}$$

$$Y_I = -2(1+R_3)\Big\{[1+R_3^2)\ z_{\pi 1} + 2/z_{\pi 1}]\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Sin}\ \theta_c$$

$$+ [(1+R_3^2)\ y_{c1}/R_3 + 2R_3/y_{c1}]\ \mathrm{Sin}\ \theta_c\ \mathrm{Cos}\ \theta_\pi\Big\} \quad (3.87)$$

$$X_{1R} = 2(1-R_3^2)(1-\mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi) + [(1+R_3)^2(R_3 z_{\pi 1}/y_{c1}$$

$$-y_{c1}/R_3 z_{\pi 1}) + (1-R_3^4)\ y_{c1}\ z_{\pi 1}/R_3]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi \quad (3.88)$$

$$X_{1I} = -2(1+R_3)\Big\{(z_{\pi 1}-1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c$$

$$-(y_{c1} - 1/y_{c1})\ R_3\ \mathrm{Sin}\ \theta_c\ \mathrm{Cos}\ \theta_\pi\Big\} \quad (3.89)$$

$$X_{2R} = -2(1-R_3^2)\ (1-\mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi) + [(1+R_3)^2(R_3 z_{\pi 1}/y_{c1}$$

$$- y_{c1}/R_3\ z_{\pi 1}) - (1-R_3^4)\ y_{c1} z_{\pi 1}/R_3]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi \quad (3.90)$$

$$X_{2I} = -2(1+R_3)[(R_3^2 z_{\pi 1} - 1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c$$

$$+ (R_3/y_{c1} - y_{c1}/R_3)\ \mathrm{Sin}\ \theta_c\ \mathrm{Cos}\ \theta_\pi] \quad (3.91)$$

$$X_{3R} = 2(1-R_3)^2\ (1-\mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi) - 2[(1+R_3^2)\ y_{c1} z_{\pi 1}$$

$$- 2R_3/y_{c1} z_{\pi 1}]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi \quad (3.92)$$

$$X_{3I} = 2(1+R_3)\ [(R_3 z_{\pi 1} - 1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c$$

$$- (R_3/y_{c1} - y_{c1})\ \mathrm{Sin}\ \theta_c\ \mathrm{Cos}\ \theta_\pi] \quad (3.93)$$

$$X_{4R} = 2(1+R_3)^2 \ (\text{Cos}\ \theta_c - \text{Cos}\ \theta_\pi) \tag{3.94}$$

$$X_{4I} = 2(1+R_3)\ [(R_3/y_{cl} + y_{cl})\ \text{Sin}\ \theta_c$$

$$- (R_3\ z_{\pi 1} + 1/z_{\pi 1})\ \text{Sin}\ \theta_\pi] \tag{3.95}$$

$$X_{5R} = 4(1+R_3)\ (\text{Cos}\ \theta_c + R_3\ \text{Cos}\ \theta_\pi) \tag{3.96}$$

$$X_{5I} = -2(1+R_3)\ [(R_3/y_{cl} + y_{cl}/R_3)\ \text{Sin}\ \theta_c + (R_3^2\ z_{\pi 1}$$

$$+ 1/z_{\pi 1})\ \text{Sin}\ \theta_\pi] \tag{3.97}$$

$$X_{6R} = -4(1+R_3)\ (R_3\ \text{Cos}\ \theta_c + \text{Cos}\ \theta_\pi) \tag{3.98}$$

$$X_{6I} = -2(1+R_3)\ [(y_{cl} + 1/y_{cl})\ R_3\ \text{Sin}\ \theta_c$$

$$+ (z_{\pi 1} + 1/z_{\pi 1})\ \text{Sin}\ \theta_\pi] \tag{3.99}$$

These expression are seen to be reducing to special cases of symmetric lines for $R_3=1$ and of homogenous lines for $\theta_c = \theta_\pi = \theta$ case.

CHAPTER 4

DESIGN CONSIDERATIONS

In earlier chapters we have derived the admittance and scattering matrix of the Asynchronous and Unsymmetric coupled lines. An analytical method to derive the expressions for design of symmetric microstrip line, a case of symmetric and Asynchronous coupled lines, is given by Haddad and Krage [5] and for an unsymmetric striplines in a homogenous dielectric medium is given by Zysman and Johnson [4].

We have to derive the relationships for designing an Asynchronous and unsymmetric pair of coupled line, that is for a case where the two parallel coupled lines are embedded on a medium which is in homogenous in the direction orthogonal to the direction of propogation. For this we have to find out the conditions for perfect matching and directivity using the scattering matrix parameters.

From the elements of S-parameter matrix obtained in Chapter 3 we can write down the conditions for perfect matching. That is, when

$$S_{11} = S_{44} = 0 = S_{22} = S_{33} \qquad (4.1)$$

From this, we get

$$X_{1R} = 0$$

$$X_{1I} = 0$$

$$X_{2R} = 0$$

$$X_{2I} = 0 \qquad (4.2)$$

One notices that the necessary and sufficient condition for these to be true is

$$R_3 = 1 \qquad (4.3)$$

$$y_{c1} = 1 \qquad (4.4)$$

and

$$z_{\pi 1} = 1 \qquad (4.5)$$

Also, as a consequence of the relation (1.29) we get

$$\frac{Y_{c1}}{Y_{c2}} = \frac{Z_{\pi 2}}{Z_{\pi 1}} = R_3 = 1$$

From equations (4.3) to (4.5) we get

$$Y_{c1} = \frac{1}{Z_o} = Y_{c2} \qquad (4.6)$$

and

$$Z_{\pi 1} = Z_{\pi 2} = Z_o \qquad (4.7)$$

This indicates that the two lines are matched to the terminations for both 'c' and 'π' modes.

In cases where the two lines are terminated by two different impedances Z_o^a and Z_o^b for ports on line 'a' and

line 'b' respectively. Expressions for A', B', C', F, H, K, G get modified and thus we get new expressions for X_{1R}, X_{1I}, X_{2R}, X_{2I} etc. We get :

$$Y_R = 2\left\{(1-\alpha\ R_3)^2 - [(1+2\alpha) + 2\alpha\ R_3 + \alpha(2+\alpha)R_3^{\ 2}]\ \mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi\right\}$$

$$+ [(1+R_3)^2\ (\alpha^2 R_3 z_{\pi 1}/y_{c1} + y_{c1}/R_3 z_{\pi 1})$$

$$+ (1+\alpha)^2\ R_3/y_{c1}\ z_{\pi 1} + (1+\alpha\ R_3^{\ 2})\ y_{c1}\ z_{\pi 1}\ /R_3]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi \tag{4.8}$$

$$Y_I = -2(1+R_3)\left\{[\alpha(1+\alpha\ R_3^{\ 2})\ z_{\pi 1} + (1+\alpha)/z_{\pi 1}]\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c\right.$$

$$\left. + [(1+\alpha\ R_3^{\ 2})\ y_{c1}/R_3 + \alpha(1+\alpha)R_3/y_{c1}]\ \mathrm{Sin}\ \theta_c\ \mathrm{Cos}\ \theta_\pi\right\} \tag{4.9}$$

$$X_{1R} = 2(1-\alpha^2 R_3^{\ 2})\ (1-\mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi) + [(1+R_3)^2\ (\alpha^2 R_3 z_{\pi 1}/y_{c1}$$

$$- y_{c1}/R_3 z_{\pi 1}) + (1-\alpha^2 R_3^{\ 4})\ y_{c1} z_{\pi 1}/R_3 \tag{4.10}$$

$$+ (1-\alpha^2)\ R_3/y_{c1}\ z_{\pi 1}]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi$$

and

$$X_{1I} = -2\alpha(1+R_3)\ [(z_{\pi 1}-1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c$$

$$- (y_{c1}-1/y_{c1})\ R_3\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c] \tag{4.11}$$

$$X_{2R} = -2(1-\alpha^2 R_3^2)\ (1-\mathrm{Cos}\ \theta_c\ \mathrm{Cos}\ \theta_\pi) + [(1+R_3)^2\ (\alpha^2 R_3\ z_{\pi 1}/y_{c1}$$

$$- y_{c1}/R_3\ z_{\pi 1}) - (1-\alpha^2 R_3^4)\ y_{c1} z_{\pi 1}/R_3$$

$$- (1-\alpha^2)\ R_3/y_{c1}\ z_{\pi 1}]\ \mathrm{Sin}\ \theta_c\ \mathrm{Sin}\ \theta_\pi \qquad (4.12)$$

and

$$X_{2I} = -2(1+R_3)\ [(\alpha^2 R_3^2 z_{\pi 1} - 1/z_{\pi 1})\ \mathrm{Sin}\ \theta_\pi\ \mathrm{Cos}\ \theta_c$$

$$+ (\alpha^2 R_3/y_{c1} - y_{c1}/R_3)\ \mathrm{Sin}\ \theta_c\ \mathrm{Cos}\ \theta_\pi] \qquad (4.13)$$

where $\alpha = Z_o^a/Z_o^b$

From these the condition for perfect matching (Equation 4.1) gives necessary and sufficient conditions as

$$\alpha = \frac{1}{R_3},\quad y_{c1} = 1 \quad \text{and } z_{\pi 1} = 1 \qquad (4.14)$$

Thus

$$\frac{y_{c1}}{y_{c2}} = \frac{z_{\pi 2}}{z_{\pi 1}} = R_3$$

which gives

$$Y_{c1} = 1/Z_o^a$$

$$Y_{c2} = 1/Z_o^b$$

$$Z_{\pi 1} = Z_o^a$$

$$Z_{\pi 2} = Z_o^b \qquad (4.15)$$

This is similar to what we have got earlier that the two coupled lines are matched to their external impedances for both 'c' and 'π' modes.

Putting these conditions in other expressions for parameters of S-matrix we get :

$$S_{12} = S_{21} = S_{34} = S_{43} = 0 \qquad (4.16)$$

while

$$S_{13} = S_{31} = S_{24} = S_{42} \neq 0 \quad \text{for } \theta_c \neq \theta_\pi \qquad (4.17)$$

The above condition implies zero coupling between the lines. This is because in a synchronous case where $\theta_c = \theta_\pi$ the coupling between the lines is introduced only by mode scattering at four ports. But in an asynchronous case relative phase rotation of one mode with respect to the other mode occurs along the physical length of the line because $v_c \neq v_\pi$. This relative phase rotation introduces power transfer as seen in condition (4.17). And, the coupling obtained so is co-directional and the condition for perfect matching is seen to be the condition for perfect directivity also.

So for a perfect matching and total directivity the 4x4 S-matrix of an asynchronous and unsymmetric pair of coupled lines becomes :

$$\begin{vmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{vmatrix} \qquad (4.18)$$

where ports 1, 2, 3 and 4 are as shown in Fig. 1.

The different expressions for various S-matrix parameters after substituting the above conditions (4.15) reduce to

$$S_{13} = -\sqrt{R_3}\,[\exp(-j\theta_\pi) - \exp(-j\theta_c)]/(1+R_3)$$

$$= [2\sqrt{R_3}\,\mathrm{Sin}\,(\frac{\theta_\pi - \theta_c}{2})/(1+R_3)]\,\exp[(j\,(\frac{\pi}{2} - \frac{\theta_\pi + \theta_c}{2})] \qquad (4.19)$$

$$S_{14} = [\exp(-j\theta_\pi) + R_3\,\exp(-j\theta_c)]/(1+R_3) \qquad (4.20)$$

$$S_{23} = [R_3\,\exp(-j\theta_\pi) + \exp(-j\theta_c)]/(1+R_3) \qquad (4.21)$$

Here taking a special case when

$$\theta_\pi = 2\,\theta_c \qquad (4.22)$$

we get

$$S_{13} = -\sqrt{R_3}\,[\exp\,(-j2\theta_c) - \exp(-j\theta_c)]/(1+R_3)$$

$$= -\sqrt{R_3}[(1-\mathrm{Cos}\,\theta_c - 2\,\mathrm{Sin}^2\theta_c) + j\,\mathrm{Sin}\,\theta_c(1-2\,\mathrm{Cos}\,\theta_c)] \qquad (4.23)$$

$$S_{14} = [\exp(-j2\theta_c) + R_3 \exp(-j\theta_c)]/(1+R_3)$$

$$= [(1+R_3 \cos \theta_c - 2 \sin^2 \theta_c) - j \sin \theta_c (R_3+2 \cos \theta_c)]/(1+R_3) \qquad (4.24)$$

and

$$S_{23} = [R_3 \exp(-j2\theta_c) + \exp(-j\theta_c)]/(1+R_3)$$

$$= [(R_3+\cos \theta_c - 2R_3 \sin^2\theta_c) - j \sin \theta_c(1+2R_3 \cos \theta_c)]/(1+R_3) \qquad (4.25)$$

From these we get the magnitude and phase of S_{23} and S_{14}. These are :

$$|S_{14}| = \sqrt{1+R_3^2+2 R_3 \cos \theta_c}/(1+R_3) \qquad (4.26)$$

and

$$|S_{23}| = \sqrt{1+ R_3^2 + 2R_3 \cos \theta_c}/(1+R_3) \qquad (4.27)$$

Also

$$\angle S_{23} = \tan^{-1} \left[\frac{\sin \theta_c (1+2 R_3 \cos \theta_c)}{(2 R_3 \sin^2 \theta_c-R_3-\cos \theta_c)}\right] \qquad (4.28)$$

and

$$\angle S_{14} = \tan^{-1} \left[\frac{\sin \theta_c(R_3+2 \cos \theta_c)}{2 \sin^2\theta_c - 1 - R_3 \cos \theta_c}\right] \qquad (4.29)$$

Thus

$$\left|S_{23}\right| = \left|S_{14}\right| \tag{4.30}$$

and

$$\angle S_{23} \neq \angle S_{14} \tag{4.31}$$

This, once again, indicates that in a perfectly matched coupler of this type coupling is mainly due to the relative phase rotation along the physical length of the line.

For designing a coupler we can relate these conditions obtained for perfect matching and directivity with the per unit length impedance and admittance parameters, y_1, y_2, y_m, z_1, z_2 and z_m defined earlier in Chapter 2.

For congruent case $R_\pi = - R_3$ and $R_c = 1$ and

$$R_3 \triangleq \frac{y_1 + y_m}{y_2 + y_m} = \frac{z_2 - z_m}{z_1 - z_m} \tag{4.32}$$

For perfect matching and directivity

$$Y_{c1} = \gamma_c(z_2-z_m)/(z_1 z_2 - z_m^2) = 1/Z_o^a \tag{4.33}$$

$$Y_{c2} = \gamma_c(z_1-z_m)/(z_1 z_2-z_m^2) = 1/Z_o^b \tag{4.34}$$

$$Z_{\pi 1} = (z_1 z_2 - z_m^2)/\gamma_\pi(z_2+R_3 z_m) = Z_o^a \tag{4.35}$$

$$Z_{\pi 2} = -(z_1 z_2 - z_m^2)/\gamma_\pi(z_1 R_3+z_m) = Z_o^b \tag{4.36}$$

From (4.33) and (4.35) we get

$$\gamma_c(z_2-z_m) = \gamma_\pi(z_2+R_3 z_m)$$

or

$$z_m = z_2(\gamma_c-\gamma_\pi)/(\gamma_c+R_3\gamma_\pi) \tag{4.37}$$

From (4.34) and (4.36)

$$z_1(\gamma_c+\gamma_\pi) = z_m(\gamma_c-\gamma_\pi/R_3)$$

or

$$z_1 = z_m(R_3\gamma_c-\gamma_\pi)/R_3(\gamma_c+\gamma_\pi) \tag{4.38}$$

From (4.33) we get

$$(z_2-z_m)\ Z_o{}^a\gamma_c = z_m{}^2 \left[\frac{R_3\gamma_c - \gamma_\pi}{R_3(\gamma_c+\gamma_\pi)}\right]\left[\frac{\gamma_c+R_3\gamma_\pi}{\gamma_c - \gamma_\pi}\right] - z_m{}^2$$

or

$$z_m = \frac{Z_o{}^a(P-1)\ \gamma_c}{(PQ - 1)} \tag{4.39}$$

where

$$P = \frac{\gamma_c + R_3\gamma_\pi}{\gamma_c - \gamma_\pi} \tag{4.40}$$

and

$$Q = \frac{R_3\gamma_c - \gamma_\pi}{R_3(\gamma_c+\gamma_\pi)} \tag{4.41}$$

So,

$$z_1 = Q\ z_m \tag{4.42}$$

$$z_2 = P\ z_m \tag{4.43}$$

Similarly, one can find out the per unit length admittance parameters y_1, y_2 and y_m from the expressions for Y_{c1}, $Z_{\pi 1}$ etc. in terms of these parameters.

We now have to relate these line admittance and impedance parameters to the actual dimension of the coupled line system viz: line widths, gap between the two lines, the dielectric constant of the substrate etc. the results which are available in literature are either for a pair of coupled lines of equal width on an inhomogenous dielectric medium or for a pair of asymmetric coupled lines in an homogenous dielectric medium.

Analytical results are not available till date relating coupling, directivity and other design parameters with various structure parameters like dielectric constant, width of the lines and gap between the two asymmetric coupled lines in an homogenous medium. The results have been reported by Akhtarzad [13] for a pair of symmetric coupled microstrip-lines using conformal mapping techniques. They have given relationships between even and odd mode impedances and the width of the lines, gap width and dielectric constant of the substrate. The results are suitable for a small range of relative dielectric constant of medium. Brynt and Weiss [12] have reported some experimental data based upon the results obtained for a pair of symmetric coupled lines on different

dielectric substrates. We have found the analytical expressions for even and odd mode impedances using the numerical techniques and the results reported by Weiss. These results are found to be quite suitable for a wide range of dielectric mediums, from relative dielectric constant of 2.0 to 10.0, within an accuracy of ± 5 percent. The expressions for Z_{oo} and Z_{oe} are :

$$Z_{oo} = \frac{120\ \pi/\sqrt{\varepsilon_r}}{\begin{aligned}[t] &[W/h + 0.7451\ (W/h)\ (1/\sqrt{\varepsilon_r}) \\ &+ 4.17363/(1+W/h)^2\ \sqrt{\varepsilon_r} \\ &+ 2.37008/(1+S/W)\sqrt{\varepsilon_r} \\ &+ 5.34568/(1+S/W)^2\sqrt{\varepsilon_r}]\end{aligned}} \tag{4.44}$$

and

$$Z_{oe} = \frac{120\ \pi/\sqrt{\varepsilon_r}}{\begin{aligned}[t] &[W/h + 0.93164\ (W/h)\ (1/\sqrt{\varepsilon_r}) \\ &+ 2.33974/(1+W/h)^2/\sqrt{\varepsilon_r} \\ &+ 0.9701\ (W/h)/(1+W/4S)/\sqrt{\varepsilon_r} \\ &+ 1.16070/(1+W/S)^2\sqrt{\varepsilon_r}]\end{aligned}} \tag{4.45}$$

where

ε_r is the effective dielectric constant

W is the width of lines

S is the gap width

and h is the height of the substrate.

The complexity of solutions increases manifold when an attempt is made to find out the analytical expressions relating design parameters or line parameters with structure parameters in an Asynchronous and unsymmetric pair of coupled lines. Although the relationships between the design and structure parameters are very important, from the practical point of view, for designing an asynchronous and unsymmetric pair of coupled line, the same is beyond the scope of this thesis.

CHAPTER 5

FILTER CONFIGURATIONS

Uniformly coupled lines circuits are used for many applications including directional couplers, impedance matching networks and filters. Most of these circuits are designed using the impedance, admittance, A B C D and other parameters for the four port coupled lines. We have already derived the properties of asymmetric uniformly coupled line structures in an inhomogenous medium in terms of self and mutual line constants characterizing the lines. These circuit parameters for coupled line four port may be used to design various structures for many known applications including filters.

Using the impedance or admittance matrix parameters given in Chapter 2 and the phase constants for two modes 'c' and 'π' with different boundary conditions we get different types of filter structures. These have been sub-divided and analysed under different categories given below.

a) Symmetric Interdigital Circuits

These are obtained either by short circuiting or open circuiting ports 2 and 4. We will first consider the short circuited case shown in Fig. 2.

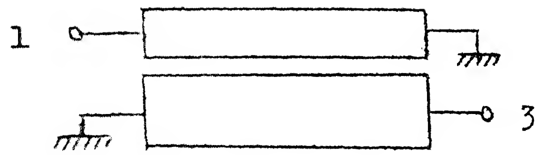

Fig. 2(a) : Symmetric Interdigital circuit for filter

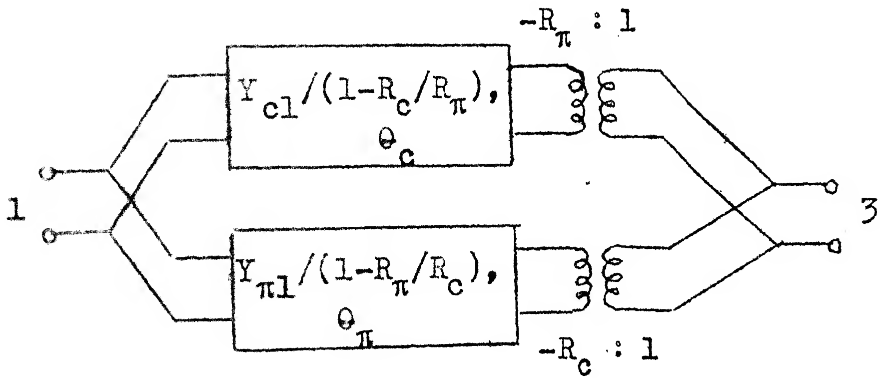

Fig. 2(b) : Equivalent Circuit

i) Short circuiting the two ports imposes the boundary condition

$$V_2 = V_4 = 0 \tag{5.1}$$

Using the Y-parameter matrix for four port coupled lines, obtained in Chapter 2, the remaining two ports are then described by the following matrix equation.

$$\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{31} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} \tag{5.2}$$

where

$$\begin{bmatrix} Y_{11} & Y_{13} \\ Y_{31} & Y_{33} \end{bmatrix}$$

$$= -j \begin{bmatrix} \dfrac{Y_{c1} \operatorname{Cot} \theta_c}{(1-R_c/R_\pi)} + \dfrac{Y_{\pi 1} \operatorname{Cot} \theta_\pi}{(1-R_\pi/R_c)} & \dfrac{Y_{c1} C_{sc} \theta_c}{R_\pi (1-R_c/R_\pi)} + \dfrac{Y_{\pi 1} C_{sc} \theta_\pi}{R_c (1-R_\pi/R_c)} \\[2ex] \dfrac{Y_{c1} C_{sc} \theta_c}{R_\pi (1-R_c/R_\pi)} + \dfrac{Y_{\pi 1} C_{sc} \theta_\pi}{R_c (1-R_\pi/R_c)} & \dfrac{Y_{c1} \operatorname{Cot} \theta_c}{R_\pi^2 (1-R_c/R_\pi)} + \dfrac{Y_{\pi 1} \operatorname{Cot} \theta_\pi}{R_c^2 (1-R_\pi/R_c)} \end{bmatrix}$$

$$= - \frac{j Y_{c1}}{(1-R_c/R_\pi)} \begin{bmatrix} \operatorname{Cot} \theta_c & C_{sc} \theta_c/R_\pi \\ C_{sc} \theta_c/R_\pi & \operatorname{Cot} \theta_c/R_\pi^2 \end{bmatrix}$$

$$- \frac{j Y_{\pi 1}}{(1-R_\pi/R_c)} \begin{bmatrix} \operatorname{Cot} \theta_c & C_{sc} \theta_\pi/R_c \\ C_{sc} \theta_\pi/R_\pi & \operatorname{Cot} \theta_\pi/R_c^2 \end{bmatrix} \tag{5.3}$$

where $\theta_c = \beta_{cl}$ and $C_{sc}\,\theta \triangleq$ Cosecant θ

and $\theta_\pi = \beta_{\pi l}$ and 'l' is the physical length of the line. Now, the Y matrix for a simple element (i.e. line of electrical length θ) is given by

$$[Y] = -j\,Y_o \begin{bmatrix} \text{Cot}\ \theta & -C_{sc}\,\theta \\ -C_{sc}\,\theta & \text{Cot}\ \theta \end{bmatrix} \qquad (5.4)$$

where Y_o is the characteristic impedance of the transmission line. A comparision of (5.4) and (5.3) suggests that the equivalent circuit for a symmetric interdigital circuits with its ports 2 and 4 short circuited is made of two transmission lines of two electrical lengths θ_c and θ_π and characteristic impedances $Y_{cl}/(1-R_c/R_\pi)$ and $Y_{\pi l}/(1-R_\pi/R_c)$ respectively joined in a parallel combination thru two transformers with impedance transformation ratios of $1/R_\pi$ and $1/R_c$. This is as shown in Fig. 2(l).

The A B C D parameters based upon these are given by

$$A = \frac{Y_{cl}\,(R_c/R_\pi - 1)\ \text{Cot}\ \theta_c + Y_{\pi l}(R_\pi/R_c - 1)\ \text{Cot}\ \theta_\pi}{\text{DEN 2}} \qquad (5.5)$$

$$B = \frac{j\,R_c R_\pi\ (1-R_\pi/R_c)\ (1-R_c/R_\pi)}{\text{DEN 2}} \qquad (5.6)$$

$$C = (AD + 1)/B \qquad (5.7)$$

$$D = \frac{R_c R_\pi [Y_{c1}(1-R_\pi/R_c) \text{ Cot } \theta_c + Y_{\pi 1}(1-R_c/R_\pi) \text{ Cot } \theta_\pi]}{\text{DEN } 2} \qquad (5.7)$$

where

$$\text{DEN } 2 = [R_c Y_{c1}(1-R_\pi/R_c) \ C_{sc}\theta_c + R_\pi Y_{\pi 1}(1-R_c/R_\pi) \ C_{sc}\theta_\pi \qquad (5.9)$$

2) In the open circuited case, ports 2 and 4 opened as shown in Fig. 3(a). This imposes the boundary condition

$$I_2 = I_4 = 0 \qquad (5.10)$$

The reamining two-port is then described by the following equation

$$\begin{vmatrix} V_1 \\ V_3 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{vmatrix} \begin{vmatrix} I_1 \\ I_3 \end{vmatrix} \qquad (5.11)$$

where

$$\begin{vmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{vmatrix} = \frac{-j \ Z_{c1}}{(1-R_c/R_\pi)} \begin{vmatrix} \text{Cot } \theta_c & R_c \ C_{sc} \ \theta_c \\ R_c C_{sc}\theta_c & R_c^2 \text{Cot } \theta_c \end{vmatrix}$$

$$- \frac{jZ_{\pi 1}}{(1-R_c/R_\pi)} \begin{vmatrix} \text{Cot } \theta_\pi & R_\pi C_{sc} \ \theta_\pi \\ R_\pi C_{sc}\theta_\pi & R_\pi^2 \text{Cot } \theta_\pi \end{vmatrix} \qquad (5.12)$$

Once again the Z matrix for a simple unit element is given by :

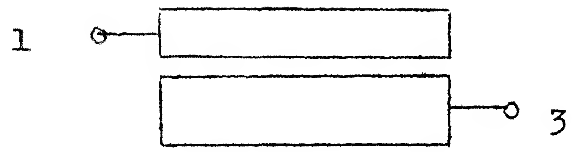

Fig. 3(a) : Symmetric Interdigital Circuit for filter

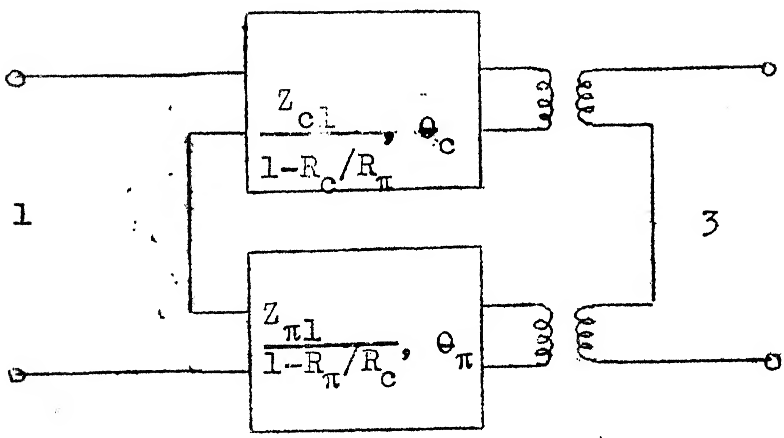

Fig. 3(b) : Equivalent circuit

$$[Z] = -j Z_o \begin{vmatrix} \text{Cot } \theta & C_{sc} \theta \\ C_{sc} \theta & \text{Cot } \theta \end{vmatrix} \tag{5.13}$$

where Z_o is the characteristic impedence of the line.

A comparision of (5.12) and (5.13) gives that the equivalent circuit consists of a series combination of a pair of transmission lines of characteristic impedances $Z_{c1}/(1-R_c/R_\pi)$ and $Z_{\pi 1}/(1-R_c/R_\pi)$ and electrical lengths of θ_c and θ_π respectively. These lines have impedance transformers of transformation ratios R_c and R_π respectively at both the ports. The equivalent circuit is as shown in Fig. 3(b). Here again, the two unit elements have different electrical lengths. The A B C D parameters may also be derived for this case and their value is given by

$$A = \frac{R_c^2 Z_{c1}(1-R_\pi/R_c) \text{ Cot } \theta_c + R_\pi^2 Z_{\pi 1}(1-R_c/R_\pi) \text{ Cot } \theta_\pi}{\text{DEN } 3} \tag{5.14}$$

$$B = (A\ D - 1)/C \tag{5.15}$$

$$C = \frac{j(1-R_c/R_\pi)\ (1-R_\pi/R_c)}{\text{DEN } 3} \tag{5.16}$$

$$D = \frac{Z_{c1}(1-R_\pi/R_c) \text{ Cot } \theta_c + Z_{\pi 1}(1-R_c/R_\pi) \text{ Cot } \theta_\pi}{\text{DEN } 3} \tag{5.17}$$

where

$$\text{DEN } 3 = [R_c Z_{c1}(1-R_\pi/R_c)\ C_{sc}\theta_c + R_\pi Z_{\pi 1}(1-R_c/R_\pi) C_{sc}\theta_\pi] \tag{5.18}$$

It is seen that for a simple case of symmetric coupled lines in a non homogenous medium ($R_c = -R_\pi = 1$ and $\theta_c = \theta_\pi$: the above two cases of interdigital filters reduce to the results obtained by Johnson [4]. Admittance matrix for the first case of short circuits at port 2 and 4 is given by

$$-\frac{jY_{cl}}{2}\begin{vmatrix} \text{Cot } \theta_c & -C_{sc} \ \theta_c \\ -C_{sc} \ \theta_c & \text{Cot } \theta_c \end{vmatrix} - \frac{j\,Y_{\pi l}}{2}\begin{vmatrix} \text{Cot } \theta_\pi & C_{sc} \ \theta_\pi \\ C_{sc} \ \theta_\pi & \text{Cot } \theta_\pi \end{vmatrix}$$

This suggests an equivalent circuit consisting of a parallel combinations two transmission lines of electrical length θ_c and θ_π characteristic impedances of $Y_{cl}/2$ and $Y_{\pi l}/2$ with one of the lines having polarity reversing transformer. This type of circuit is reported to be a band pass-filter.

Similarly in the other case with open circuits at ports 2 and 4 the results are similar to those reported by Johnson [4]. And this type of circuit is also reported to be a band pass filter.

b) Prototype Meander like Sections

3) The basic circuit is constructed by tying together terminals 3 and 4 of the four port in Fig. 1. This imposes the boundary conditions (Fig. 4(a):

$$V_3 = V_4 \quad \text{and} \quad I_3 = -I_4 \tag{5.19}$$

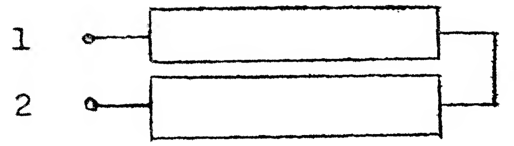

Fig. 4(a) : Meander line section for filter circuit

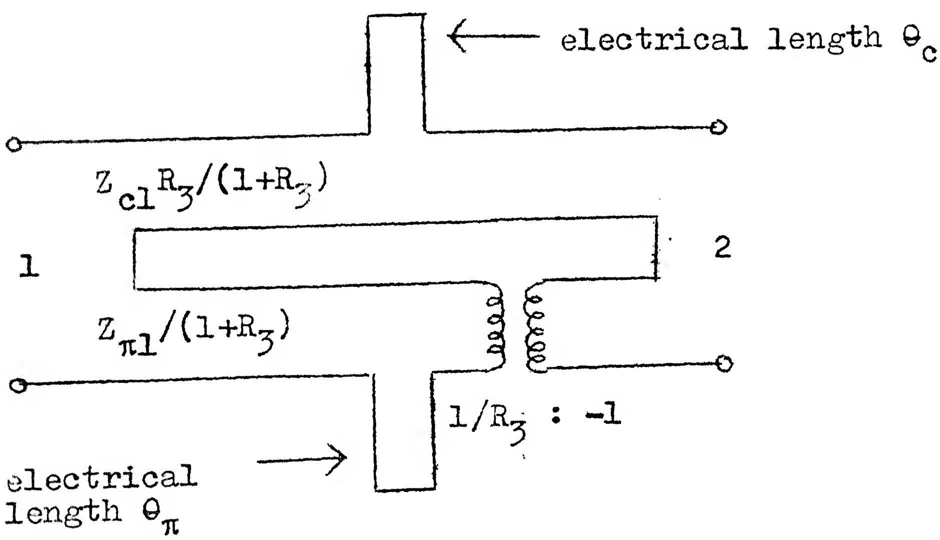

Fig. 4(b) : Equivalent circuit

Rewriting the impedance matrix of a four port coupled line structure in a way as :

$$\begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{vmatrix} = \begin{vmatrix} a & b & c & d \\ b & e & f & c \\ c & f & e & b \\ d & c & b & a \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{vmatrix} \tag{5.20}$$

where a, b, c, d, e and f the six independent entries of the 4x4 matrix. Applying the boundary conditions obtained for meander line section we get :

$$\begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} \tag{5.21}$$

where

$$Z_{11} = a + (c-d)^2/(2b-e-a) \tag{5.22}$$

$$Z_{12} = b + (c-d)\ (f-c)/(2b-e-a) = Z_{21} \tag{5.23}$$

and

$$Z_{22} = e + (f-c)^2/(2b-e-a) \tag{5.24}$$

Using the expressions for a, b, c etc. from Chapter 2, we get :

$$a = [R_3 Z_{c1} \text{ Coth } \gamma_{c1}^{*} + Z_{\pi 1} \text{ Coth } \gamma_{\pi 1}^{*}]/(R_3+1) \qquad (5.25)$$

$$(c-d) = -Z_{\pi 1}/\text{Sinh } \gamma_{\pi 1}$$

or

$$(c-d)^2 = Z_{\pi 1}^2 \, /\text{Sinh}^2 \, \gamma_{\pi 1} \qquad (5.26)$$

$$(2b-c-a) = -(1+R_3) \, Z_{\pi 1} \text{ Coth } \gamma_{\pi 1} \qquad (5.27)$$

Once again

$$(f-c) = R_3 Z_{\pi 1}/\text{Sinh } \gamma_{\pi 1}$$

or

$$(f-c)^2 = R_3^2 Z_{\pi 1}^2/\text{Sinh}^2 \, \gamma_{\pi 1} \qquad (5.28)$$

From expressions in (5.25) to (5.28) we get :

$$Z_{11} = [R_3 Z_{c1} \text{ Coth } \gamma_{c1} - Z_{\pi 1} \tanh \gamma_{\pi 1}]/(R_3+1) \qquad (5.29)$$

$$Z_{12} = R_3[Z_{c1} \text{ Coth } \gamma_{c1} - Z_{\pi 1} \tanh \gamma_{\pi 1}]/(R_3+1) = Z_{21} \qquad (5.30)$$

and

$$Z_{22} = R_3[Z_{c1} \text{ Coth } \gamma_{c1} + R_3 Z_{\pi 1} \tanh \gamma_{\pi 1}]/(R_3+1) \qquad (5.31)$$

From these we get the impedance matrix of a two port meander line lossless section as:

* $\gamma_{\pi 1} = \gamma_{\pi} l$; * $\gamma_{c1} = \gamma_c l$

$$[Z] = -\frac{jZ_{c1}R_3\text{Cot}\ \theta_c}{(1 + R_3)}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$-\frac{jZ_{\pi 1}\ \tan\ \theta_\pi}{(1 + R_3)}\begin{bmatrix}1 & -R_3\\-R_3 & R_3^2\end{bmatrix} \tag{5.32}$$

Comparing this with impedance matrix of a unit element (5.13) an equivalent circuit is as given in Fig. 4(b). The equivalent circuit consists of two stubs with one of them cascaded with a polarity reversing transformer. The A B C D matrix parameters are as given below :

$$A = \frac{(R_3 Z_{c1}\ \text{Coth}\ \gamma_{c1} - Z_{\pi 1}\ \tanh\ \gamma_{\pi 1})}{R_3\ \cdot\ \text{DEN 4}} \tag{5.33}$$

$$B = \frac{(R_3+1)}{R_3\ \cdot\ \text{DEN 4}} \tag{5.34}$$

$$C = \frac{-(Z_{c1}\ \text{Coth}\ \gamma_{c1} + R_3\ Z_{\pi 1}\ \tanh\ \gamma_{\pi 1})}{\text{DEN 4}} \tag{5.35}$$

and $\quad B = (AD + 1)/C \quad (5.36)$

where

$$\text{DEN 4} = (Z_{c1}\ \text{Coth}\ \gamma_{c1} - Z_{\pi 1}\ \tanh\ \gamma_{\pi 1}) \tag{5.37}$$

In an homogenous dielectric case, the above structure with two equal lines is a microwave type-C section [16] which is an all pass circuit. In an homogenous case, however, the even and odd mode elements resonate at some frequencies creating a stop band. The band pass behaviour of a similar case has been confirmed experimentally [4].

(c) Comb line circuits :

4) This circuit is constructed by open circuiting terminals 3 and 4 in Fig. 1. The resulting structure is shown in Fig. 5(a) and this imposes the boundary conditions

$$I_3 = I_4 = 0 \tag{5.38}$$

The impedance matrix of this structure is

$$[Z] = \frac{-j\, Z_{c1} \operatorname{Cot} \theta_c}{(1 - R_c/R_\pi)} \begin{bmatrix} 1 & R_c \\ R_c & R_c^{\,2} \end{bmatrix} - \frac{jZ_{\pi 1} \operatorname{Cot} \theta_\pi}{(1 - R_\pi/R_c)} \begin{bmatrix} 1 & R_\pi \\ R_\pi & R_\pi^{\,2} \end{bmatrix} \tag{5.39}$$

Comparing this with that of a unit element a simple equivalent circuit is suggested as shown in Fig. 5(b). The A B C D parameters of this case are given as

$$A = \frac{Z_{c1}(1-R_\pi/R_c) \operatorname{Cot} \theta_c + Z_{\pi 1}(1-R_c/R_\pi) \operatorname{Cot} \theta_\pi}{\text{DEN } 5} \tag{5.40}$$

$$B = (A\, D - 1)/C \tag{5.41}$$

$$C = \frac{j(1-R_c/R_\pi)\,(1-R_\pi/R_c)}{\text{DEN } 5} \tag{5.42}$$

$$D = -\frac{R_c^2 Z_{c1} \text{ Cot } \theta_c (1-R_\pi/R_c) + R_\pi^2 Z_{\pi 1} \text{ Cot } \theta_\pi (1-R_c/R_\pi)}{\text{DEN } 5} \tag{5.43}$$

where

$$\text{DEN } 5 = R_c Z_{c1}(1-R_\pi/R_c) \text{ Cot } \theta_c + R_\pi Z_{\pi 1} \text{ Cot } \theta_\pi (1-R_c/R_\pi) \tag{5.44}$$

5) In the dual comb line circuit where ports 3 and 4 are short circuited as shown in Fig. 6(a) the boundary conditions imposed are :

$$V_3 = V_4 = 0 \tag{5.45}$$

From this, the relationship between the other two ports is given as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{5.46}$$

where

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = -\frac{j\, Y_{c1} \text{ Cot } \theta_c}{(1 - R_c/R_\pi)} \begin{bmatrix} 1 & -1/R_\pi \\ -1/R_\pi & 1/R_\pi^2 \end{bmatrix} - \frac{j\, Y_{\pi 1} \text{ Cot } \theta_\pi}{(1 - R_\pi/R_c)} \begin{bmatrix} 1 & -1/R_c \\ -1/R_c & 1/R_c^2 \end{bmatrix} \tag{5.47}$$

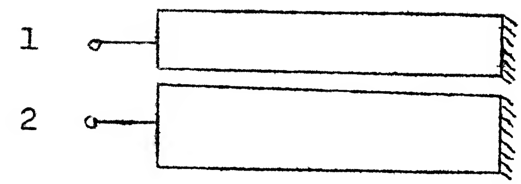

Fig. 6(a) : Dual comb line circuit for filter configuration.

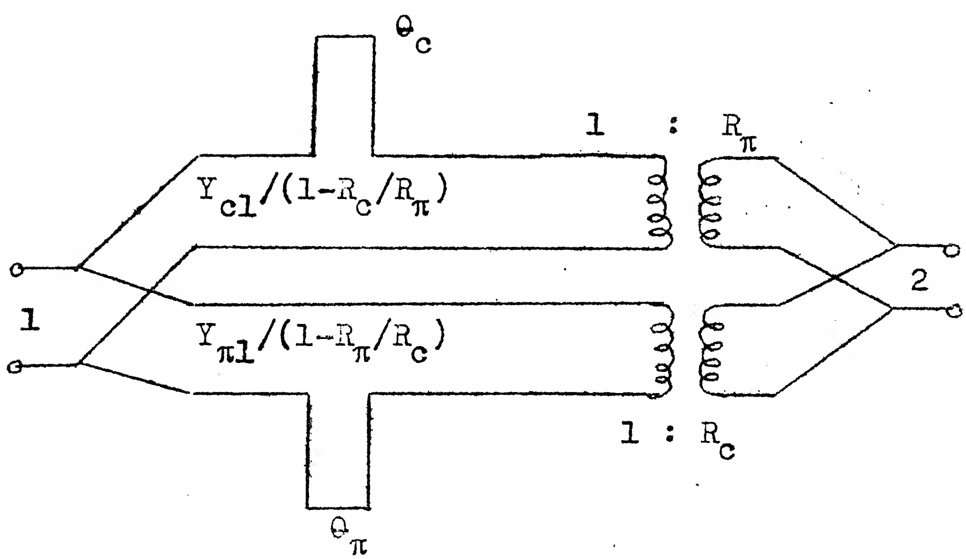

Fig. 6(b) : Equivalent circuit representation

The equivalent circuit for this is as shown in Fig. 6(b) with two stubs and two impedance transformers. From these Y-parameters the A B C D parameters which are obtained are as:

$$A = \frac{(R_c/R_\pi - 1)\, Y_{c1} \operatorname{Cot} \theta_c + (R_\pi/R_c - 1)\, Y_{\pi 1} \operatorname{Cot} \theta_\pi}{\text{DEN } 6} \tag{5.48}$$

$$B = -\frac{j\, R_\pi R_c (1 - R_\pi/R_c)\,(1 - R_c/R_\pi)}{\text{DEN } 6} \tag{5.49}$$

$$C = (A\, D + 1)/B \tag{5.50}$$

$$D = -\frac{R_\pi R_c [(1 - R_\pi/R_c)\, Y_{c1} \operatorname{Cot} \theta_c + (1 - R_c/R_\pi)\, Y_{\pi 1} \operatorname{Cot} \theta_\pi]}{\text{DEN } 6} \tag{5.51}$$

where $\text{DEN } 6 = R_c(1 - R_\pi/R_c) Y_{c1} \operatorname{Cot} \theta_c + R_\pi (1 - R_c/R_\pi) Y_{\pi 1} \operatorname{Cot} \theta_\pi$ (5.52)

(d) Other Circuits

There are two more symmetrical circuits which have been analysed here.

6) The open circuited symmetric structure is as shown in Fig. 7(a) and this imposes the boundary conditions given by

$$I_2 = I_3 = 0 \tag{5.53}$$

Using the Z matrix given in Chapter 2 for a 4 port coupled line case we get the impedance matrix for this two port case given by

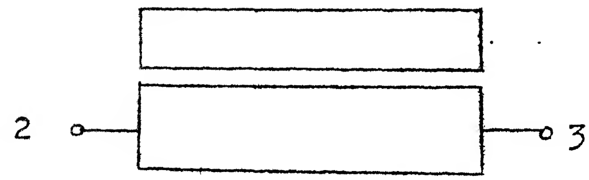

Fig. 7(a) : Open circuited symmetric structure

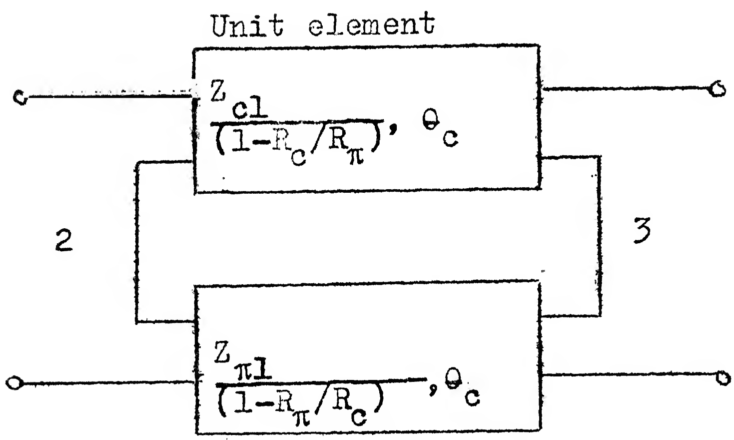

Fig. 7(b) : Equivalent circuit representation

$$[Z] = \begin{bmatrix} Z_{11} & Z_{14} \\ Z_{41} & Z_{44} \end{bmatrix} = -\frac{j\, Z_{c1}}{(1-R_c/R_\pi)} \begin{bmatrix} \mathrm{Cot}\ \theta_c & C_{sc}\ \theta_c \\ C_{sc}\ \theta_c & \mathrm{Cot}\ \theta_c \end{bmatrix}$$

$$- \frac{j\, Z_{\pi 1}}{(1-R_\pi/R_c)} \begin{bmatrix} \mathrm{Cot}\ \theta_\pi & C_{sc}\ \theta_\pi \\ C_{sc}\ \theta_\pi & \mathrm{Cot}\ \theta_\pi \end{bmatrix} \qquad (5.54)$$

The equivalent circuit for this is as shown in Fig. 7(b) and the A B C D parameters are as given below :

$$A = -\frac{[Z_{c1}(1-R_\pi/R_c)\ \mathrm{Cot}\ \theta_c + Z_{\pi 1}(1-R_c/R_\pi)\ \mathrm{Cot}\ \theta_\pi]}{\mathrm{DEN}\ 7} = -D \qquad (5.55)$$

$$(5.56)$$

$$B = (A\ D - 1)/C \qquad (5.57)$$

$$C = \frac{j(1-R_c/R_\pi)\ (1-R_\pi/R_c)}{\mathrm{DEN}\ 7} \qquad (5.58)$$

$$D = -A$$

where

$$\mathrm{DEN}\ 7 = Z_{c1}(1-R_\pi/R_c)\ C_{sc}\ \theta_c + Z_{\pi 1}(1-R_c/R_\pi)\ C_{sc}\ \theta_\pi \qquad (5.59)$$

7) Short circuited symmetric structure is as shown in Fig. 8(a) where the ports 2 and 3 are short circuited. This imposes the boundary conditions given by

$$V_2 = V_3 = 0 \qquad (5.60)$$

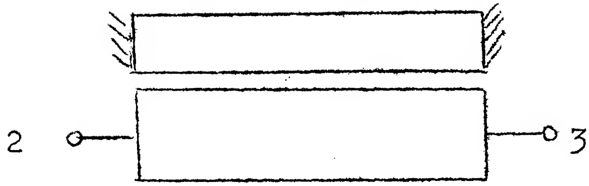

Fig. 8(a) : Short circuited symmetric structure for filter configuration

Unit element

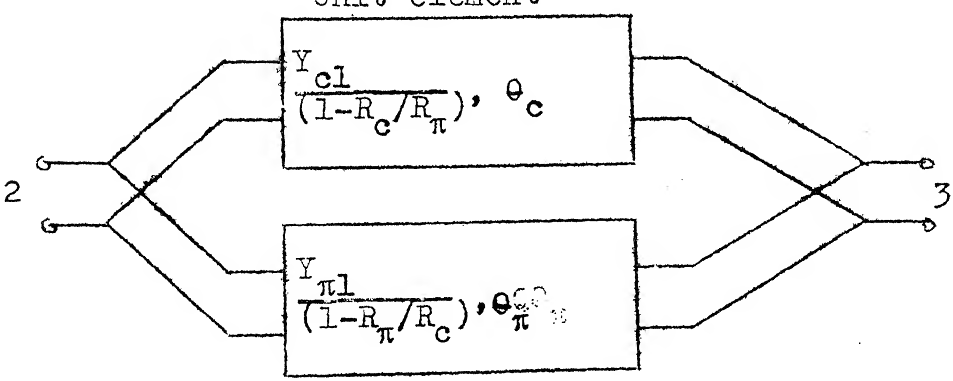

Fig. 8(b) : Equivalent circuit

and the 2 ports are represented by the matrix equation

$$\begin{bmatrix} I_1 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{41} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \end{bmatrix} \tag{5.61}$$

where

$$\begin{bmatrix} Y_{11} & Y_{14} \\ Y_{41} & Y_{44} \end{bmatrix} = -\frac{j\, Y_{c1}}{(1-R_c/R_\pi)} \begin{bmatrix} \mathrm{Cot}\ \theta_c & -\,C_{sc}\ \theta_c \\ -C_{sc}\ \theta_c & \mathrm{Cot}\ \theta_c \end{bmatrix} - \frac{j\, Y_{\pi 1}}{(1-R_\pi/R_c)} \begin{bmatrix} \mathrm{Cot}\ \theta_\pi & -C_{sc}\ \theta_\pi \\ -C_{sc}\ \theta_\pi & \mathrm{Cot}\ \theta_\pi \end{bmatrix} \tag{5.62}$$

The equivalent circuit is, thus, as shown in Fig. 8(b) and A B C D parameters are :

$$A = \frac{(1-R_\pi/R_c)\ Y_{c1}\ \mathrm{Cot}\ \theta_c + Y_{\pi 1}(1-R_c/R_\pi)\ \mathrm{Cot}\ \theta_\pi}{\mathrm{DEN}\ 8} \tag{5.63}$$

$$B = -\frac{j(1 - R_c/R_\pi)\ (1-R_\pi/R_c)}{\mathrm{DEN}\ 8} \tag{5.64}$$

$$C = (A\ D + 1)/B \tag{5.65}$$

$$\text{and } D = -A \tag{5.66}$$

where

$$\mathrm{DEN}\ 8 = Y_{c1}(1-R_\pi/R_c)\ C_{sc}\theta_c + Y_{\pi 1}(1-R_c/R_\pi)\ C_{sc}\ \theta_\pi \tag{5.67}$$

(e) Unsymmetric Networks :

There are three such cases discussed below. Since the calculation of parameters for a general case of Asymmetric and unsynchronous coupled lines becomes complex, the filter parameters out of a special case of congurent symmetry [10] is discussed. Also, the unsymmetric networks do not readily lend themselves equivalent circuit representations.

8) First of such cases is when port 2 is short circuited and port 4 is open circuited as shown in Fig. 9 . This imposes, the boundary conditions :

$$
\begin{aligned}
I_4 &= 0 \\
V_2 &= 0
\end{aligned}
\tag{5.68}
$$

and using 4x4 matrix given by equation (5.20) we get the 2-port case represented by

$$
\begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} \tag{5.69}
$$

where

$$Z_{11} = (a - b^2/e) \tag{5.70}$$

$$Z_{13} = Z_{31} = (c - b\ f/e) \tag{5.71}$$

$$Z_{33} = (e - f^2/e) \tag{5.72}$$

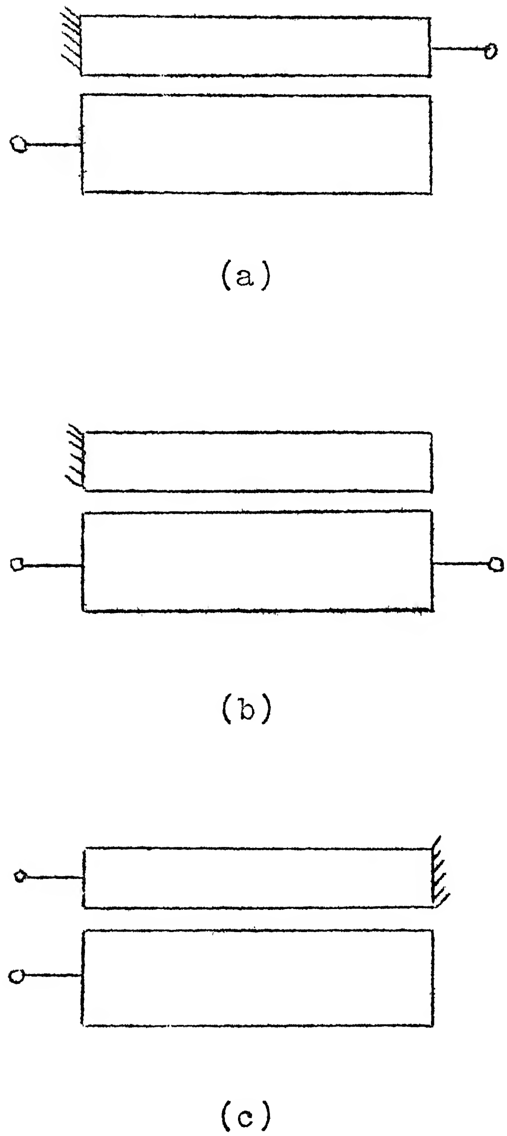

Fig. 9 : Unsymmetric networks for filter configurations

Using the congruent symmetry condition in a, b, c and other parameters we obtain :

$$Z_{11} = \frac{(1+R_3)(Z_{\pi 1} \operatorname{Coth} \gamma_{c1})\ (Z_{\pi 1} \operatorname{Coth} \gamma_{\pi 1})}{\text{DEN } 9} \tag{5.73}$$

$$Z_{33} = \frac{R_3[Z_{c1}^2 + R_3^2 Z_{\pi 1}^2 + 2R_3 Z_{c1} Z_{\pi 1}(\operatorname{Coth} \gamma_{c1} \operatorname{Coth} \gamma_{\pi 1} - \operatorname{Csc} \gamma_{c1} \operatorname{Csc} \gamma_{\pi 1})]}{(1+R_3) \quad \text{DEN } 9} \tag{5.74}$$

$$Z_{13} = \frac{R_3 Z_{c1} Z_{\pi 1}(\operatorname{Cosh} \gamma_{\pi 1} - \operatorname{Cosh} \gamma_{c1})}{(\operatorname{Sinh} \gamma_{\pi 1} \operatorname{Sinh} \gamma_{c1}) \cdot \text{DEN } 9} \tag{5.75}$$

where

$$\text{DEN } 9 = Z_{c1} \operatorname{Coth} \gamma_{c1} + R_3 Z_{\pi 1} \operatorname{Coth} \gamma_{\pi 1} \tag{5.76}$$

From these, we can get the A B C D parameters :

$$A = \left(\frac{1+R_3}{R_3}\right) \left(\frac{\operatorname{Cosh} \gamma_{\pi 1} \operatorname{Cosh} \gamma_{c1}}{\operatorname{Cosh} \gamma_{\pi 1} - \operatorname{Cosh} \gamma_{c1}}\right) \tag{5.77}$$

$$B = (A\ D + 1)/C \tag{5.78}$$

$$C = \frac{Z_{c1} \operatorname{Coth} \gamma_{c1} + R_3\ Z_{\pi 1} \operatorname{Coth} \gamma_{c1}}{R_3 Z_{c1} Z_{\pi 1} \left[\dfrac{\operatorname{Cosh} \gamma_{\pi 1} - \operatorname{Cosh} \gamma_{c1}}{\operatorname{Sinh} \gamma_{\pi 1} \operatorname{Sinh} \gamma_{c1}}\right]} \tag{5.79}$$

$$D = -\frac{Z_{cl}^{2}+R_3^{2}Z_{\pi l}^{2}(\text{Sinh}\ \gamma_{\pi l}\ \text{Sinh}\ \gamma_{cl}) + 2R_3 Z_{cl} Z_{\pi l}(\text{Cosh}\ \gamma_{cl}\ \text{Cosh}\ \gamma_{\pi l} - 1)}{(1+R_3)\ Z_{cl} Z_{\pi l}} \tag{5.80}$$

a) In another case of unsymmetric network port 2 is a short circuited and port 3 is open circuited Fig. 9 . Thus, imposing the conditions

$$V_2 = 0$$

$$\text{and } I_3 = 0 \tag{5.81}$$

Where the impedance matrix of the two port circuit is given by

$$\begin{bmatrix} V_1 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{14} \\ Z_{41} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_4 \end{bmatrix} \tag{5.82}$$

and

$$Z_{11} = \frac{(1+R_3)(Z_{cl}\ \text{Coth}\ \gamma_{cl})\ (Z_{\pi l}\ \text{Coth}\ \gamma_{\pi l})}{\text{DEN 9}} \tag{5.83}$$

$$Z_{44} = \frac{R_3 \text{Sinh}\gamma_{cl}\,\text{Sinh}\gamma_{\pi l}(Z_{cl}^{2}+Z_{\pi l}^{2})+Z_{cl}Z_{\pi l}(1+R_3^{2})\text{Cosh}\gamma_{\pi l}\,\text{Cosh}\gamma_{cl}+2R}{\text{Sinh}\ \gamma_{cl}\ \text{Sinh}\ \gamma_{\pi l}\ (1+R_3)\ \text{DEN 9}} \tag{5.84}$$

$$Z_{14} = \frac{R_3 Z_{cl} Z_{\pi l}\ \text{Cosh}\ \gamma_{\pi l} + Z_{cl} Z_{\pi l}\ \text{Cosh}\ \gamma_{cl}}{\text{Sinh}\ \gamma_{cl}\ \text{Sinh}\ \gamma_{\pi l}\cdot \text{DEN 9}} = Z_{41} \tag{5.85}$$

where DEN 9 is as defined earlier. The corresponding A B C D parameters are given by

$$A = \frac{(1+R_3)\ \text{Cosh}\ \gamma_{c1}\ \text{Cosh}\ \gamma_{\pi 1}}{\text{Den 10}} \tag{5.86}$$

$$B = (A\ D + 1)/C$$

$$C = \frac{\text{Sinh}\ \gamma_{c1}\ \text{Sinh}\ \gamma_{\pi 1}\ (Z_{c1}\ \text{Coth}\ \gamma_{c1} + R_3 Z_{\pi 1}\ \text{Coth}\ \gamma_{\pi 1})}{Z_{c1}\ Z_{\pi 1}\ \text{Den 10}} \tag{5.87}$$

$$D = \frac{R_3(Z_{c1}^2 + Z_{\pi 1}^2)\ (\text{Sinh}\ \gamma_{c1}\ \text{Sinh}\ \gamma_{\pi 1}) + (1+R_3^2)\ Z_{c1} Z_{\pi 1}\ (\text{Cosh}\ \gamma_{c1}\ \text{Cosh}\ \gamma_{\pi 1}) + 2R_3}{(1+R_3)\ Z_{c1}\ Z_{\pi 1}\ \text{Den 10}} \tag{5.88}$$

where

$$\text{Den 10} = R_3\ \text{Cosh}\ \gamma_{\pi 1} + \text{Cosh}\ \gamma_{c1} \tag{5.89}$$

10) The last of the casesamong two port unsymmetric networks is as shown in Fig. 9 where port 3 is short circuited and port 4 in Fig. 1 is open circuited with this we get the boundary conditions

$$I_4 = 0 = V_3 \tag{5.90}$$

and impedance matrix parameters given as :

Z_{11} is same as Z_{44} given in case 9, equation (5.84).

$$Z_{22} = \frac{R_3[Z_{c1}{}^2+R_3{}^2Z_{\pi 1}{}^2+2R_3Z_{c1}Z_{\pi 1}(\mathrm{Coth}\ \gamma_{\pi 1}\ \mathrm{Coth}\ \gamma_{c1}-C_{sc}\gamma_{c1}C_{sc}\gamma_{\pi 1}]}{(1+R_3)\ \mathrm{DEN}\ 9} \tag{5.91}$$

$$Z_{21} = \left(\frac{R_3}{1+R_3}\right) \frac{Z_{c1}{}^2-R_3{}^2Z_{\pi 1}{}^2+Z_{c1}Z_{\pi 1}(R_3-1)(\mathrm{Coth}\gamma_{\pi 1}\mathrm{Coth}\gamma_{c1}-C_{sc}\gamma_{\pi 1}C_{sc}\gamma_{c1}}{\mathrm{DEN}\ 9} \tag{5.92}$$

where DEN 9 is as defined earlier. The expressions for A, B, C, D parameters come out to be very complex and are given below :

$$A = \frac{1}{R_3}\left[\frac{R_3(Z_{c1}{}^2+Z_{\pi 1}{}^2)+(1+R_3{}^2)Z_{c1}Z_{\pi 1}\mathrm{Coth}\gamma_{\pi 1}\mathrm{Coth}\gamma_{c1}+2R_3C_{sc}\gamma_{c1}C_{sc}\gamma_{\pi 1}}{\mathrm{DEN}\ 11}\right. \tag{5.93}$$

$$B = (A\ D + 1)/C \tag{5.94}$$

$$C = \frac{Z_{c1}\mathrm{Coth}\ \gamma_{c1} + R_3\ Z_{\pi 1}\ \mathrm{Coth}\ \gamma_{\pi 1}}{\mathrm{DEN}\ 11} \tag{5.95}$$

$$D = \frac{Z_{c1}{}^2+R_3{}^2Z_{\pi 1}{}^2+2R_3Z_{c1}Z_{\pi 1}(\mathrm{Coth}\ \gamma_{\pi 1}\mathrm{Coth}\gamma_{c1}-C_{sc}\gamma_{c1}C_{sc}\gamma_{\pi 1})}{\mathrm{DEN}\ 11} \tag{5.96}$$

where

$$\mathrm{DEN}\ 11 = Z_{c1}{}^2-R_3{}^2Z_{\pi 1}{}^2+Z_{c1}Z_{\pi 1}(R_3-1)(\mathrm{Coth}\ \gamma_{\pi 1}\mathrm{Coth}\ \gamma_{c1}-C_{sc}\gamma_{\pi 1}C_{sc}\gamma_{c1}) \tag{5.97}$$

This indicates that the characteristics of various coupled line circuits embedded in an inhomogenous medium (such as suspended substrates) with the two lines of different impedances differ markedly from those in homogenous medium or from the symmetric cases. But all the above expressions obtained in this chapter are seen to be reducing to the expressions for special cases of symmetric lines in nonhomogenous medium [4] and of homogenous cases [2] . Experimental results for these special cases are reported in literature.

CHAPTER 6

CONCLUSIONS AND DISCUSSIONS

In case of asymmetric, uniform coupled lines, in a non-homogenous medium e.g. suspended substrates, microstrip lines and others, analysis can be done in terms of the line properties for two independent modes of excitation. The characteristics of two modes which travel simultaneously in a general case, i.e. propogation constants and characteristic impedances, are derived in terms of per unit length parameters of the coupled line structure, like, series impedances, shunt admittances and mutual impedance and admittance of the two lines. From these 4 port network matrices are obtained and it is seen that there are six independent entries of these 4x4 matrices.

In a large class of nonsymmetrical coupled lines in a nonhomogenous propogation medium, a voltage even mode and a current odd mode are found to be the fundamental uncoupled modes of the structure [10]. Like in case of inhomogenous dielectric medium with symmetric lines, both the modes travel independently, one at a particular excitation, for each mode, at different velocities. These two modes have been characterized by having wave voltages of equal magnitude and

same phase and wave currents of equal magnitude and opposite phase respectively. The condition for the existence of these simple modes called congurence condition, is $C_a/C_b = (L_b-M)/(L_a-M)$. This condition is satisfied by non symmetrical line in a homogenous medium, and implies for a nonhomogenous dielectric, that the ratio of the two line to ground capacitances C_a and C_b is same in the filled and in the empty structure.

These circuit parameters characterizing the four port may be used to design various structures including filters, couplers and matching networks. It is observed that a condition for perfect matching gives perfect directivity also in a 4 port coupled line structure with congurent symmetry. Also, in this case under perfect matching at the ports, the coupling is mainly co-directional and power transfer is mainly due to relative phase rotation due to unequal mode velocities. An attempt has been made to relate the coupling and dielectric parameters to the per unit length parameters of the two coupled lines.

It is suggested that for making full use of the work done out here, calculation have to be done relating the various line length parameters and the dimensions of coupled mode structure. In turn, this will give relations between

properties of coupled mode structures in terms of gap width, effective dielectric constant and strip widths. This work is slightly involved and can be done either analytically using conformal mapping techniques or numerically using finite difference techniques.

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